Math 3325 **Homework 3** **Spring 2007**

The assignment is due at the beginning of class on February 23, 2007.

Problem 1 (10 points) Let A, B and C be arbitrary sets. We define $A \triangle B := (A \setminus B) \cup (B \setminus A)$. Prove or disprove:

1. $A \triangle B = B \triangle A$.
2. $(A \triangle B) \triangle C = A \triangle (B \triangle C)$.

Problem 2 (10 points) Let A and B be arbitrary sets. Prove or disprove:

1. $\mathcal{P}(A \cap B) \subseteq \mathcal{P}(A) \cap \mathcal{P}(B)$.
2. $\mathcal{P}(A) \cap \mathcal{P}(B) \subseteq \mathcal{P}(A \cap B)$.

Problem 3 (10 points) Let A be a proper subset of some set U, and let $x \in U \setminus A$. Let $\mathcal{B}$ consist of all sets of the form $C \cup \{x\}$ with $C \in \mathcal{P}(A)$, in other words $\mathcal{B} = \{C \cup \{x\} \mid C \in \mathcal{P}(A)\}$. Show that

1. $\mathcal{P}(A \cup \{x\}) = \mathcal{P}(A) \cup \mathcal{B}$.
2. $\mathcal{P}(A) \cap \mathcal{B} = \emptyset$.

Problem 4 (10 points) Given two real numbers $a < b$, the open interval (a, b) is defined to be the set $\{x \in \mathbb{R} \mid (a < x) \wedge (x < b)\}$.

For $n \in \mathbb{N}$, let A_n be the open interval $(\frac{1}{2}, \frac{1}{2} + \frac{1}{n})$. Find $\bigcup_{n \in \mathbb{N}} A_n$ and $\bigcap_{n \in \mathbb{N}} A_n$. Confirm your conjectures by proofs.

Problem 5 (10 points) For each rational number $q \in \mathbb{Q}$, let $B_q = \{x \in \mathbb{R} \mid x \neq q\}$. Find $\bigcup_{q \in \mathbb{Q}} B_q$ and $\bigcap_{q \in \mathbb{Q}} B_q$. Confirm your conjectures by proofs.