

The assignment is due at the beginning of class on March 26, 2007.

Problem 1 (10 points) Let R be a relation from the set A to the set B , and S be a relation from the set B to the set C . Prove that $(S \circ R)^{-1} = R^{-1} \circ S^{-1}$.

Problem 2 (10 points) Let R and S be two relations on \mathbb{R} : $R = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid y < x^2\}$ and $S = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid y = 2x - 1\}$. Find $S \circ R$ and $R \circ S$.

Problem 3 (10 points) Let R be a relation from the set A to the set B , and S be a relation from the set B to the set C .

1. Prove or disprove: $\text{Dom}(S \circ R) \subseteq \text{Dom}(R)$.
2. Prove or disprove: $\text{Rng}(S \circ R) \subseteq \text{Rng}(S)$.

Problem 4 (10 points) Let R be a relation from A to B . For an element $b \in B$ define the set $R_b := \{a \in A \mid (a, b) \in R\}$. Show

$$\bigcup_{b \in B} R_b = \text{Dom } R.$$

Problem 5 (10 points) Define a relation R on \mathbb{R} as follows: $a R b$ if $a - b$ is irrational. Prove or disprove: R is (a) reflexive, (b) symmetric, (c) transitive.