

The assignment is due at the beginning of class on April 23, 2007.

Problem 1 (10 points) 1. Find a function whose domain is the set of real numbers \mathbb{R} and whose range is the set of rational numbers \mathbb{Q} .

2. Find a function whose domain is the set of natural numbers \mathbb{N} and whose range is the set of integers \mathbb{Z} .

Problem 2 (10 points) Prove or disprove: If f and g are functions from A to B , then $f \cap g$ is a function.

Problem 3 (10 points) 1. Find functions $f : B \rightarrow C$, $g : A \rightarrow B$ and $h : A \rightarrow B$ such that $f \circ g = f \circ h$, yet $g \neq h$.

2. Suppose $f : A \rightarrow B$ is a function with $\text{range}(f) = B$. Prove or disprove: If $g : B \rightarrow C$ and $h : B \rightarrow C$ satisfy $g \circ f = h \circ f$, then $g = h$.

Problem 4 (10 points) Let $f : A \rightarrow B$ and $g : C \rightarrow D$ be two functions. Define

$$f \times g = \{(a, c), (b, d) \mid (a, b) \in f \wedge (c, d) \in g\}.$$

Show that $f \times g$ is a function from $A \times C$ to $B \times D$. Find an explicit expression for $f \times g$.

Problem 5 (10 points) Let A and B be sets with m and n elements respectively. What is the probability that a relation from A to B chosen at random is a function?