Math 5370 **Homework 1** **Fall 2007**

The assignment is due at the beginning of class on September 17, 2007.

Problem 1 (10 points) Negate the following statement: "All dogs have three legs, or there is a cat with two tails."

Problem 2 (10 points) In each case, give an example, or explain why such an example cannot exist:

- Is there a predicate $A(x, y)$ such that the statement $\forall x\ \exists y:\ A(x, y)$ is true, while the statement $\exists y\ \forall x:\ A(x, y)$ is false?
- Is there a predicate $A(x, y)$ such that the statement $\exists y\ \forall x:\ A(x, y)$ is true, while the statement $\forall x\ \exists y:\ A(x, y)$ is false?

Problem 3 (10 points) A clothing store advertises: "For every customer we have a rack of clothes that fit."

- Write the statement above using quantifier(s) and predicate(s).
- Negate the sentence using quantifier(s) and predicate(s).
- Write the negation in the form of an English sentence.

Problem 4 (10 points) You have seen how to generate compound statements using the four connectives $\neg, \vee, \wedge$ and $\Rightarrow$. This problem addresses the question whether all four connectives are necessary.

- Use a truth table to show that $A \Rightarrow B$ is equivalent to $\neg(A \wedge \neg B)$.
- Show that $A \vee B$ can be written using only the connectives $\neg$ and $\wedge$.

Thus the two connectives $\neg$ and $\wedge$ suffice to generate all compound statements. It is possible to further reduce to only one connective, albeit a different one: Let us define the new connective NOR by setting

$$A \text{ NOR } B \iff \neg(A \vee B).$$

- Show that the four compound statements $\neg A,\ A \vee B,\ A \wedge B$ and $A \Rightarrow B$ can be written using only the NOR-connective.