

*The assignment is due at the beginning of class on October 17, 2007.*

**Problem 1 (10 points)** Prove for all natural numbers  $n \geq 5$ :  $2^n > n^2$ .

**Problem 2 (10 points)** Let  $n \in \mathbb{N}$ . Conjecture a formula for the expression

$$a_n = \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{n \cdot (n+1)}$$

and prove your conjecture by induction.

**Problem 3 (10 points)** Use induction to show the following theorem: If a set has  $n$  elements, then its power set has  $2^n$  elements. (See Homework 2 Problem 3.)

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For a natural number  $n$ , let  $\mathcal{D}_n$  denote the set of the divisors of  $n$ . For example,  $\mathcal{D}_{42} = \{1, 2, 3, 6, 7, 14, 21, 42\}$  and  $\mathcal{D}_{12} = \{1, 2, 3, 4, 6, 12\}$ . For  $m, n \in \mathbb{N}$  let  $m \sqcap n$  denote the greatest common divisor of  $n$  and  $m$ , and  $m \sqcup n$  their least common multiple. For instance  $6 \sqcap 4 = 2$  and  $6 \sqcup 4 = 12$ . It turns out that  $\mathcal{D}_{42}$  with these two operations  $\sqcap$  and  $\sqcup$  forms a Boolean Algebra, while  $\mathcal{D}_{12}$  does **not**.

**Problem 4 (10 points)** Verify Boolean Algebra Laws 3, 4 and 5 for  $\mathcal{D}_{42}$ .

**Problem 5 (10 points)** 1. Show that  $\mathcal{D}_{12}$  does not form a Boolean Algebra.

2. Conjecture for which values of  $n$  the set  $\mathcal{D}_n$  forms a Boolean Algebra.