Math 5370 **Homework 6** **Fall 2007**

The assignment is due at the beginning of class on October 24, 2007.

Problem 1 (10 points) Show that 6 divides $n^3 - n$ for all $n \in \mathbb{N}$.

Problem 2 (10 points)

1. Show that every positive integer can be written as the sum of (one or more) distinct powers of 2. (Examples: $8 = 2^3$, $25 = 2^4 + 2^3 + 2^0$.)
2. Can every positive integer be written as the sum of (one or more) distinct powers of 3?

Problem 3 (10 points) An element m in a set of real numbers A is called *maximum* of A, if $m \geq a$ for all $a \in A$. Show that every finite non-empty set of real numbers has a maximum.

Problem 4 (10 points) In class you saw a proof of the result that every polynomial (with real coefficients) can be written as a product of (one or more) irreducible polynomials, using the *Principle of Complete Induction*. Give an alternative proof using the *Well-ordering Principle*.

Problem 5 (10 points) A *prime number* is a natural number only divisible by 1 and itself; the first few prime numbers are $2, 3, 5, 7, 11, 13, \ldots$ Use a suitable induction principle to show that every natural number other than 1 can be written as a product of (one or more) prime numbers.