Math 5370 **Homework 8** **Fall 2007**

The assignment is due at the beginning of class on November 12, 2007.

Problem 1 (10 points) Consider the following relation on the set $A = \{1, 2, 3, 4, 5, 6\}$:

$$R = \{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6),(1,2),(1,4),(2,1),(2,4),(4,1),(4,2),(3,6),(6,3)\}.$$

Find the partition generated by R.

Problem 2 (10 points) Let R be a relation on $\mathbb{N}$ defined by

$$(m, n) \in R \Leftrightarrow m^2 + n^2 \text{ is even.}$$

1. Show that R is an equivalence relation.
2. Find all distinct equivalence classes of this relation.

Problem 3 (10 points) Let R and S be two equivalence relations on a non-empty set X. Prove or disprove:

1. $R \cap S$ is an equivalence relation.
2. $R \cup S$ is an equivalence relation.

Problem 4 (10 points) A relation R on a non-empty set X is called *reverse-transitive* if

$$(a, b) \in R \wedge (b, c) \in R \Rightarrow (c, a) \in R \text{ for all } a, b, c \in X.$$

Show that a relation R on a non-empty set X is an equivalence relation if and only if it is reflexive and reverse-transitive.

Problem 5 (10 points) Consider the following relation R defined on a Boolean Algebra $\mathcal{A}$:

$$(P, Q) \in R \Leftrightarrow P \sqcup Q = Q$$

Prove or disprove: R is (a) reflexive, (b) symmetric, (c) transitive.