Math 5370 **Homework 10** **Fall 2007**

The assignment is due at the beginning of class on December 5, 2007.

Problem 1 (10 points)

1. Find a function whose domain is the set of real numbers $\mathbb{R}$ and whose range is the set of rational numbers $\mathbb{Q}$.
2. Find a function whose domain is the set of natural numbers $\mathbb{N}$ and whose range is the set of integers $\mathbb{Z}$.

Problem 2 (10 points) Prove or disprove: If f and g are functions from A to B, then $f \cap g$ is a function.

Problem 3 (10 points)

1. Find functions $f : B \to C$, $g : A \to B$ and $h : A \to B$ such that $f \circ g = f \circ h$, yet $g \neq h$.
2. Suppose $f : A \to B$ is a function with $range(f) = B$. Prove or disprove: If $g : B \to C$ and $h : B \to C$ satisfy $g \circ f = h \circ f$, then $g = h$.

Problem 4 (10 points) Let $f : A \to B$ and $g : C \to D$ be two functions. Define

$$f \times g = \{((a, c), (b, d)) \mid (a, b) \in f \ \wedge \ (c, d) \in g\}.$$

Show that $f \times g$ is a function from $A \times C$ to $B \times D$. Find an explicit expression for $f \times g$.

Problem 5 (10 points) Let A and B be sets with m and n elements respectively. What is the probability[1] that a relation from A to B chosen at random is a function?

[1]The probability is the ratio of the number of distinct functions from A to B to the number of all distinct relations form A to B.