Problem 1 (5 points) Give a precise description of the Euclidean distance d in the coordinate plane as a function of points (x_1, y_1) and (x_2, y_2) in $\mathbb{R}^2$; that is, identify the domain A and range B and a rule for the distance function $d : A \to B$.

Each of the two points (x_1, y_1) and (x_2, y_2) is an element of $\mathbb{R}^2$. Therefore the domain of the function d is $\mathbb{R}^2 \times \mathbb{R}^2$.

The distance of two points in the Euclidean plane is always non-negative. Therefore the range of the function d is $\{x \in \mathbb{R} \mid x \geq 0\} = \mathbb{R}^+ \cup \{0\}$.

By the Pythagorean Theorem, the rule for the function d is given by

$$d((x_1, y_1), (x_2, y_2)) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}.$$

Problem 2 (5 points) A typical problem in elementary algebra:

An airplane makes a round trip where the one-way distance is 1000 km. On the out-leg the plane faces a headwind of 50 km/h, while on the return trip there is a tailwind of 50 km/h. If the speed of the plane in still air is 400 km/h, what is the total time of the trip?

Sketch a rough graph of a function giving the total time for the round trip in terms of wind speed as the wind speed varies from 0 to 400 km/h.

Let T denote the travel time for the round trip in hours (h) and w the wind speed in km/h. Then the total travel time as a function of wind speed is given by

$$T(w) = \frac{1000}{400 - w} + \frac{1000}{400 + w}.$$

The problem gives the domain as $[0, 400)$, a suitable co-domain is $\mathbb{R}$. Actually, the range of the function $T(w)$ is the interval $[5, \infty)$.