

Let \mathcal{K} denote the set of 2×2 matrices of the form $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$, where $a, b \in \mathbb{R}$.

- a. Show: If A and B are elements in \mathcal{K} , then $A + B \in \mathcal{K}$.
- b. Show: If A and B are elements in \mathcal{K} , then $A \cdot B \in \mathcal{K}$.
- c. Show that $A \cdot B = B \cdot A$ holds for all elements $A, B \in \mathcal{K}$.
- d. Show: If the matrix $A = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ in \mathcal{K} satisfies $a^2 + b^2 \neq 0$, then A has a matrix inverse. What is A^{-1} ? Does A^{-1} always lie in \mathcal{K} ? (Note that the identity matrix is an element of \mathcal{K} .)
- e. Show that we can identify \mathcal{K} with \mathbb{C} , i.e., find a bijection $f : \mathcal{K} \rightarrow \mathbb{C}$ such that $f(A + B) = f(A) + f(B)$ and $f(A \cdot B) = f(A) \cdot f(B)$. (This is easier than it sounds.)
- f. Under this identification, find the element I in \mathcal{K} that corresponds to i . Compute $I \cdot I$.
- g. Under this identification, what is the “conjugate” of \mathcal{K} , what is the “modulus” of \mathcal{K} ?
- h. The identification suggests a way to compute $\frac{a + bi}{c + di}$ if $c + di \neq 0$. Explain!
- i. Compute $\frac{2 + 3i}{1 - 2i}$.