Midterm **Analysis** **Fall 2000**

No books, notes etc. are allowed.

Problem 1. Let (X, d) be a metric space.

1. Define: "x_0 is an accumulation point of the set A in X."
2. Let A be a subset of X, and $x_0 \notin A$. Show that x_0 is an accumulation point of A if and only if A contains a sequence which converges to x_0.

Problem 2. Let (X, d) be a metric space, and A be a subset of X.

1. Define the concepts of "interior of A" (int A) and "closure of A" (cl A).
2. Show that $\mathrm{int}(X \setminus A) = X \setminus (\mathrm{cl}\ A)$.

Problem 3.

1. Give the definitions for "compactness" and for "sequential compactness" of a metric space.
2. Show that a compact metric space is sequentially compact. (You may use that a compact metric space is complete.)

Problem 4.

1. Give the definition for a "connected set" in a metric space.
2. Show: Let (X, d) be a metric space. Let A_1 and A_2 be two connected subsets of X, such that $A_1 \cap A_2 \neq \emptyset$. Show that $A_1 \cup A_2$ is connected.

Problem 5. Let (X, d) be a metric space, and let $A \subseteq X$. Show: If A is connected and contains more than one point, then every point of A is an accumulation point of A.