Math 4303 **Worksheet #1** **January 19, 2010**

Consider the following classic problem.

> A substance is 99% water. Some water evaporates, leaving a substance that is 98% water. How much of the water has evaporated?

a. **Getting the initial answer:** Solve the given problem.

b. **The classic status of the problem:** Based on your answer to Part a, indicate why you think this is called a classic problem.

c. **A numerical approach:** In answering Part a you may have used algebra, setting up unknowns and solving equations. Answer Part a again, this time using no algebra, but only concrete, numerical reasoning. (For example, if the original substance had 1 unit of solid stuff and 99 units water, the evaporated substance still has 1 unit of solid stuff, so . . .)

d. **A diagrammatic approach:** Solve the problem yet again, this time using a diagram as your basic reasoning tool. (For example, a simple rectangle can be divided into two regions representing the water and the solid in the original substance. A different rectangle can represent the evaporated substance . . .)

e. **Generalizing the problem:** Solve the problem again, but this time replace the numerical values 99% and 98% in the statement of the problem with general parameters. (This is a first step to generalizing the problem.)

f. **A functions approach:** The algebraic solution from Part e is fully general, yet it is not fully revealing about why evaporating half the water has lowered the proportion of water only about 1%. In a sense, this solution is too general to focus on the essentials of this problem. Solve the problem again, but this time keep the specific numerical value 1% of the drop in water content part of the solution, and express the proportion of water evaporated as a function of the original proportion of water in the substance.

g. **Another functions approach:** The approach outlined in the discussion of Part f is not the only functions approach possible. You can also express the proportion of water as a function of the absolute amount of water in the substance, letting the fixed amount of solute $S = 1$. Try this.