Math 4303 **Worksheet #3** **March 2010**

Let $\mathcal{K}$ denote the set of 2×2 matrices of the form $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$, where $a, b \in \mathbb{R}$.

a. Show: If A and B are elements in $\mathcal{K}$, then $A + B \in \mathcal{K}$.

b. Show: If A and B are elements in $\mathcal{K}$, then $A \cdot B \in \mathcal{K}$.

c. Show that $A \cdot B = B \cdot A$ holds for all elements A, $B \in \mathcal{K}$.

d. Show: If the matrix $A = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ in $\mathcal{K}$ satisfies $a^2 + b^2 \neq 0$, then A has a matrix inverse. What is A^{-1}? Does A^{-1} always lie in $\mathcal{K}$? (Note that the identity matrix Id is an element of $\mathcal{K}$.)

e. Show that we can identify $\mathcal{K}$ with $\mathbb{C}$, i.e., find a bijection $f : \mathcal{K} \to \mathbb{C}$ such that $f(A + B) = f(A) + f(B)$ and $f(A \cdot B) = f(A) \cdot f(B)$. (This is easier than it sounds.)

f. Under this identification, find the element I in $\mathcal{K}$ that corresponds to i in $\mathbb{C}$. Compute $I \cdot I$.

g. Under this identification, what is the "conjugate" of an element in $\mathcal{K}$, what is the "modulus" of an element in $\mathcal{K}$?