Math 3341 **Homework 1** **Fall 2010**

The assignment is due at the beginning of class on September 13, 2010.

Problem 1 (20 points) A non-empty set is called bounded if it is bounded from above and bounded from below.

1. Write down definitions for "bounded from below" and for greatest lower bound (infimum).
2. Show that the following three statements are equivalent:
   - Every set of real numbers that is bounded from above has a supremum.
   - Every set of real numbers that is bounded from below has an infimum.
   - Every bounded set of real numbers has both an infimum and a supremum.

Problem 2 (10 points) Show that the sequence $\left(\frac{2n^2-1}{n^2+4}\right)_{n=1}^{\infty}$ converges to 2.

Problem 3 (10 points)

1. Show: If $(|a_n|)_{n=1}^{\infty}$ converges to 0, then $(a_n)_{n=1}^{\infty}$ converges to 0.
2. Prove or give a counterexample: If $(a_n)_{n=1}^{\infty}$ converges, then $(|a_n|)_{n=1}^{\infty}$ converges.
3. Prove or give a counterexample: If $(|a_n|)_{n=1}^{\infty}$ converges, then $(a_n)_{n=1}^{\infty}$ converges.

Problem 4 (10 points) Let A be a bounded set of real numbers. Show that there is a sequence $(a_n)_{n=1}^{\infty}$ of elements in A that converges to $\sup A$.