**Math 3341** **Homework 2** **Fall 2010**

***The assignment is due at the beginning of class on September 22, 2010.***

**Problem 1 (10 points)** Let $\mathcal{A}(S)$ denote the set of accumulation points of a set $S$. Suppose $S$ and $T$ are sets of real numbers.

1. Show: If $S \subseteq T$, then $\mathcal{A}(S) \subseteq \mathcal{A}(T)$.
2. Show that $\mathcal{A}(S \cup T) = \mathcal{A}(S) \cup \mathcal{A}T)$.
3. Is it true that $\mathcal{A}(S \cap T) = \mathcal{A}(S) \cap \mathcal{A}(T)$? (Give a proof or provide a counterexample.)

**Problem 2 (15 points)** Find the set of accumulation points of the set $\left\{ \dfrac{1}{m+n} \ \mid \ m, n \in \mathbb{N} \right\}$.

**Problem 3 (10 points)** Problem 23 on page 55.

**Problem 4 (15 points)** For a real number $x$ the floor function $\lfloor . \rfloor$ is defined as

$$\lfloor x \rfloor = \max\{m \in \mathbb{Z} \mid m \leq x\}.$$

Similarly, the ceiling function $\lceil . \rceil$ is defined as

$$\lceil x \rceil = \min\{m \in \mathbb{Z} \mid m \geq x\}.$$

1. Show that the sequence $(a_n)$ defined by

$$a_n = \frac{\lfloor \pi \rfloor + \lfloor 2\pi \rfloor + \lfloor 3\pi \rfloor + \cdots + \lfloor n\pi \rfloor}{n^2}$$

   converges to $\dfrac{\pi}{2}$.

2. Investigate the convergence behavior of the sequence $(b_n)$ with

$$b_n = \frac{\lceil x \rceil + \lceil 4x \rceil + \lceil 9x \rceil + \cdots + \lceil n^2 x \rceil}{n^3}$$

   for arbitrary $x \in \mathbb{R}$.