Math 3341 **Homework 6** **Fall 2010**

The assignment is due at the beginning of class on November 24, 2010.

Problem 1 (10 points) Let $f : \mathbb{R} \to \mathbb{R}$ be continuous on $\mathbb{R}$, and assume that for all $\varepsilon > 0$ there is an $N > 0$ such that $|f(x)| < \varepsilon$ for all x satisfying $|x| > N$. Show that f is uniformly continuous on $\mathbb{R}$.

Problem 2 (15 points) Let $f : [a, b] \to \mathbb{R}$ be a function. We say f satisfies $(*)$ if there is an $M > 0$ such that $|f(x) - f(y)| \leq M \cdot |x - y|$ for all $x, y \in [a, b]$.

1. Show: If f satisfies $(*)$, then f is uniformly continuous on $[a, b]$.
2. Let $f : [0, 1] \to \mathbb{R}$ be given by $f(x) = \sqrt{x}$. Show that f does not satisfy $(*)$. (Note that f is uniformly continuous on $[0, 1]$.)

Problem 3 (10 points) Let S be a set of real numbers satisfying the following property: Every sequence in S has a subsequence that converges to an element in S.

1. Show that S is bounded.
2. Show that S is closed.

Problem 4 (15 points) For $D \subseteq \mathbb{R}$, let $\mathcal{A}(D)$ denote the set of accumulation points of D. Let $\overline{D} = D \cup \mathcal{A}(D)$.

1. Show that $\overline{D}$ is closed.
2. Assume that $D \neq \emptyset$ is bounded from above. Show that $\overline{D}$ is bounded from above. Moreover, show that $\sup D = \sup \overline{D}$.