Math 3341 **Homework 1** **Spring 2011**

The assignment is due at the beginning of class on February 3, 2011.

Problem 1 (10 points) Suppose x, y, x', y' are positive real numbers such that $\frac{x}{y} < \frac{x'}{y'}$. Show that

$$\frac{x}{y} < \frac{x+x'}{y+y'} < \frac{x'}{y'}.$$

Problem 2 (10 points) Let A and B be two sets of real numbers.

1. Assume that $\emptyset \neq A \subseteq B$ and that B is bounded from above. Show that A is bounded from above, and moreover that $\sup A \leq \sup B$.

2. State a similar theorem for sets that are bounded from below. (No proof required.)

Problem 3 (10 points) Let $A = \{x \in \mathbb{Q} \mid x^2 \leq 3\}$. Show that A is bounded from above, but that A has no maximum.

Problem 4 (10 points) A non-empty set is called *bounded* if it is bounded from above and bounded from below.

Assume that every bounded set of real numbers has both an infimum and a supremum. Show that this implies the Completeness Axiom for the Real Numbers.
Hint: This is not as easy as it may look at first glance.

Problem 5 (10 points) Using the **definition** of convergence, show that the sequence $\left(\frac{2n^2-1}{n^2+4}\right)_{n=1}^{\infty}$ converges to 2.

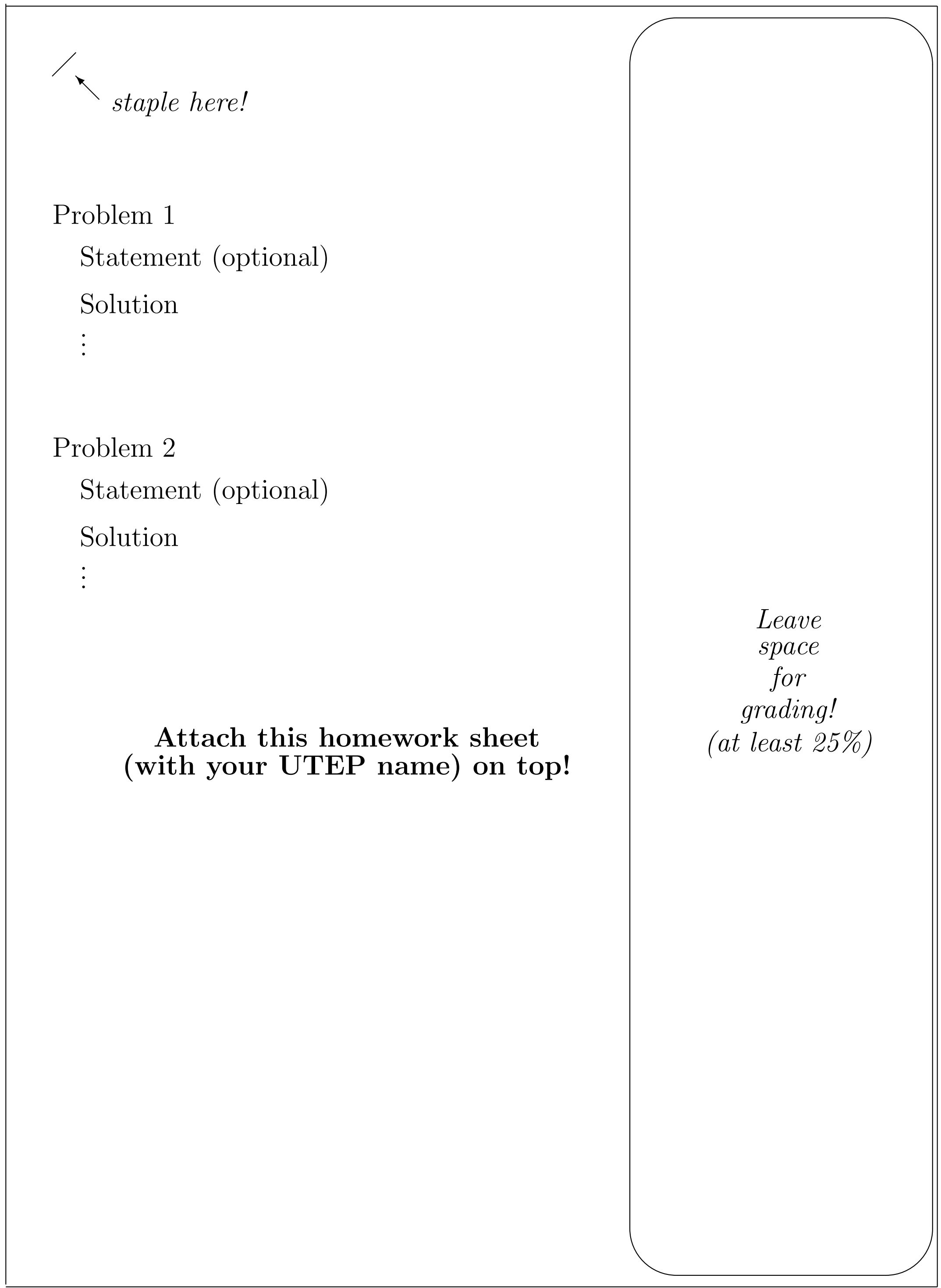

Figure 1: Homework Layout