Math 3341 **Homework 3** **Spring 2011**

The assignment is due at the beginning of class on March 8, 2011.

Problem 1 (15 points) A sequence (a_n) is called *proper*, if $a_n \neq a_m$ for all $n \neq m$.

1. Let (a_n) be a proper bounded sequence. Show: if $\{a_n \mid n \in \mathbb{N}\}$ has **exactly one** accumulation point, then (a_n) converges.
2. Show that 1. fails if we omit the hypothesis that the sequence is bounded.
3. Show that 1. fails if we omit the hypothesis that the sequence is proper.

Problem 2 (10 points) Let $(a_n)_{n=1}^{\infty}$ be a Cauchy sequence, and let $\varphi : \mathbb{N} \to \mathbb{N}$ be a one-to-one function. Show that the sequence $\left(a_{\varphi(n)}\right)_{n=1}^{\infty}$ is Cauchy.

Problem 3 (10 points) Let A be a non-empty set that is bounded from below, and let m denote its infimum. Show: $m \in A$ or m is an accumulation point of A.

Problem 4 (15 points) Problem 31 on page 56.

Extra Credit Problem 5 (15 points) Consider the following two properties:

(1) Every non-empty set that is bounded from above has a supremum.

(2) Every Cauchy sequence converges.

Show that (2)$\Rightarrow$(1).