Math 3341 **Homework 4** **Spring 2011**

The assignment is due at the beginning of class on March 24, 2011.

Problem 1 (15 points) Let $f : \mathbb{R} \to \mathbb{R}$ be a function such that $\lim_{x \to 0} f(x)$ exists. Define $g : \mathbb{R} \to \mathbb{R}$ for $x \in \mathbb{R}$ by

$$g(x) = \max\{f(x), 0\}.$$

Show that $\lim_{x \to 0} g(x)$ exists. How are the two limits related?

Problem 2 (10 points) Problem 26 on page 80.

Problem 3 (10 points) Let $f : [a, b] \to \mathbb{R}$ be an increasing function. Show that $\lim_{x \to a} f(x)$ exists. What can you say about the relationship between this limit and $f(a)$?

Problem 4 (15 points) Let $f : [0, 1] \to \mathbb{R}$ be a bounded function. Define $\hat{f} : [0, 1] \to \mathbb{R}$ for $x \in [0, 1]$ by

$$\hat{f}(x) := \inf\{f(t) \mid 0 \leq t \leq x\}.$$

Show that $\hat{f}$ is a decreasing function. Then show that $\hat{f}$ has a limit at $x_0 \in (0, 1)$, if f itself has a limit at x_0 and $\lim_{x \to x_0} f(x) = f(x_0)$.