Math 3341 **Homework 6** **Spring 2011**

The assignment is due at the beginning of class on May 3, 2010.

Problem 1 (10 points) Given a set S and $x \in S$, x is called an *interior point* of S if there is a neighborhood of x that is contained in S. Show that the set

$$T = \{x \in S \mid x \text{ is an interior point of } S\}$$

is an open set.

Moreover, show: If U is an open set contained in S, then U is also contained in T.

Problem 2 (15 points) Use the **definition** of compactness to show the following:

1. The union of finitely many compact sets is compact.
2. Let A be a non-empty set. If $\{C_\alpha \mid \alpha \in A\}$ is a collection of compact sets, then $\bigcap_{\alpha \in A} C_\alpha$ is compact.

Problem 3 (15 points) Show: If $X \subseteq \mathbb{R}$ is open **and** closed, then $X = \mathbb{R}$ or $X = \emptyset$.

Problem 4 (10 points) Let $f : [0,1] \to [0,1]$ be a continuous function. Show that there is an $x \in [0,1]$ such that $f(x) = x$.