Find all $x$ such that

$$\log x - \log(2 + \sqrt{x}) = 1$$

**Solution I.**

$$\log x - \log(2+\sqrt{x}) = 1$$
$$\Leftrightarrow \log\left(\frac{x}{2+\sqrt{x}}\right) = 1 \Leftrightarrow \frac{x}{2+\sqrt{x}} = 10$$
$$\Leftrightarrow x - 10\sqrt{x} - 20 = 0$$

Set $\sqrt{x} = y$:

$$y^2 - 10y - 20 = 0,$$

so

$$y = 5 \pm \sqrt{25+20} = 5 \pm \sqrt{45}.$$

Consequently $x = (5 \pm \sqrt{45})^2$.

**Solution II.**

$$\log x - \log(2+\sqrt{x}) = 1$$
$$\Leftrightarrow \log\left(\frac{x}{2+\sqrt{x}}\right) = 1 \Leftrightarrow \frac{x}{2+\sqrt{x}} = 10$$
$$\Leftrightarrow x - 20 = 10\sqrt{x}$$

Square both sides:

$$x^2 - 40x + 400 = 100x \Leftrightarrow x^2 - 140x + 400 = 0$$

Thus $x = 70 \pm \sqrt{4900 - 400} = 70 \pm \sqrt{4500}$

**Solution III.**

$$\log x - \log(2 + \sqrt{x}) = 1$$
$$\Leftrightarrow \log\left(\frac{x}{2 + \sqrt{x}}\right) = 1 \Leftrightarrow \frac{x}{2 + \sqrt{x}} = 10$$
$$\Leftrightarrow x - 10\sqrt{x} - 20 = 0$$

We want to factor this as $(\sqrt{x} - a)(\sqrt{x} - b)$. This requires $ab = -20$ and $a + b = 10$. Solving this system for $a$ and $b$ yields:

$$a(10 - a) = -20 \Leftrightarrow a^2 - 10a - 20 = 0.$$

So $a = 5 \pm \sqrt{25 + 20} = 5 \pm \sqrt{45}$ and then $b = 5 \mp \sqrt{45}$. Thus $\sqrt{x} = 5 + \sqrt{45}$, discarding the negative solution;

So $x = (5 + \sqrt{45})^2$.