Boolean Algebras

Statements and their connectives on the one hand, and sets and set connectives on the other hand behave somewhat analogously. The English mathematician *George Boole* (1815–1864) made this idea precise by describing what he called "algebra of logic". Today we use the name "Boolean Algebra" in his honor instead:

A *Boolean Algebra* is a set $\mathcal{B}$ together with two "connectives" $\sqcap$ and $\sqcup$ satisfying the following properties:

1. Closure Laws:
   (a) If A and B are two elements in $\mathcal{B}$, then $A \sqcap B$ is also an element in $\mathcal{B}$.
   (b) If A and B are two elements in $\mathcal{B}$, then $A \sqcup B$ is also an element in $\mathcal{B}$.
2. Commutative Laws:
   (a) $A \sqcap B = B \sqcap A$ for all elements A and B in $\mathcal{B}$.
   (b) $A \sqcup B = B \sqcup A$ for all elements A and B in $\mathcal{B}$.
3. Distributive Laws:
   (a) $A \sqcap (B \sqcup C) = (A \sqcap B) \sqcup (A \sqcap C)$ for all elements A, B and C in $\mathcal{B}$.
   (b) $A \sqcup (B \sqcap C) = (A \sqcup B) \sqcap (A \sqcup C)$ for all elements A, B and C in $\mathcal{B}$.
4. There are elements $N \in \mathcal{B}$ (called the *null element*) and $O \in \mathcal{B}$ (the *one element*) such that
   (a) $A \sqcap N = N$ and $A \sqcap O = A$ for all elements A in $\mathcal{B}$.
   (b) $A \sqcup O = O$ and $A \sqcup N = A$ for all elements A in $\mathcal{B}$.
5. For every element A in $\mathcal{B}$ there is an element B in $\mathcal{B}$ such that $A \sqcap B = N$ and $A \sqcup B = O$.