UNIV 1301 **Worksheet 2** **Fall 2011**

Problem 1 Show that $A \wedge B$ is equivalent to $\neg(\neg A \vee \neg B)$.

Problem 2 Show that $(A \wedge (A \Rightarrow B)) \Rightarrow B$.

The Greeks called this a "syllogism". Here is a classical example: *Socrates is human. All humans are mortal. Thus Socrates is mortal.* (A is "x is human", B is "x is mortal".)

Problem 3 Show that $A \Rightarrow B$ is equivalent to $\neg B \Rightarrow \neg A$.

Problem 4 Let n be a natural number. Show: If n^2 is even, then n is even. (Hint: Use the previous problem.)

Problem 5 Show that the propositional forms $A \Rightarrow B$ and $\neg A \vee B$ are equivalent.

Problem 6 Show that the propositional forms $\neg(A \Rightarrow B)$ and $A \wedge \neg B$ are equivalent.

Problem 7 Problem 5 shows that the connective "$\Rightarrow$" is superfluous in the following sense: Every instance of "$\Rightarrow$" in a propositional form can be replaced by using "$\neg$" and "$\vee$" instead. Similarly Problem 1 shows that one can replace occurrences of "$\wedge$" by using "$\neg$" and "$\vee$" instead.

Does one really need two connectives to represent the other familiar connectives? No, one connective suffices, albeit a different one:

We define a new connective NAND by declaring that P NAND Q is equivalent to $\neg(P \wedge Q)$. Show that the propositional forms $P \vee Q$, $P \wedge Q$, $\neg P$ and $P \Rightarrow Q$ all have equivalent forms containing only instances of the "NAND"-connective.