Math 3341 **Homework 1** **Fall 2011**

The assignment is due at the beginning of class on September 7, 2011.

Problem 1 (10 points) Suppose a set S of real numbers has a maximum, call it m. Show that m is also the supremum of S.

Problem 2 (10 points) Let $A = \{x \in \mathbb{Q} \mid x^2 \leq 3\}$. Show that A is bounded from above, but that A has no maximum.

Problem 3 (10 points) Let A and B be two sets of real numbers.

1. Assume that $\emptyset \neq A \subseteq B$ and that B is bounded from above. Show that A is bounded from above, and moreover that $\sup A \leq \sup B$.
2. State a similar theorem for sets that are bounded from below. (No proof required.)

Problem 4 (10 points) A non-empty set is called *bounded* if it is bounded from above and bounded from below.

Assume that every bounded set of real numbers has both an infimum and a supremum. Show that this implies the Completeness Axiom for the Real Numbers.
Hint: This is not as easy as it may look at first glance.

Problem 5 (10 points)

1. Show the triangle inequality:

$$|x + y| \leq |x| + |y| \text{ holds for all } x, y \in \mathbb{R}.$$

2. Show the reverse triangle inequality:

$$|x - y| \geq \big|\,|x| - |y|\,\big| \text{ holds for all } x, y \in \mathbb{R}.$$

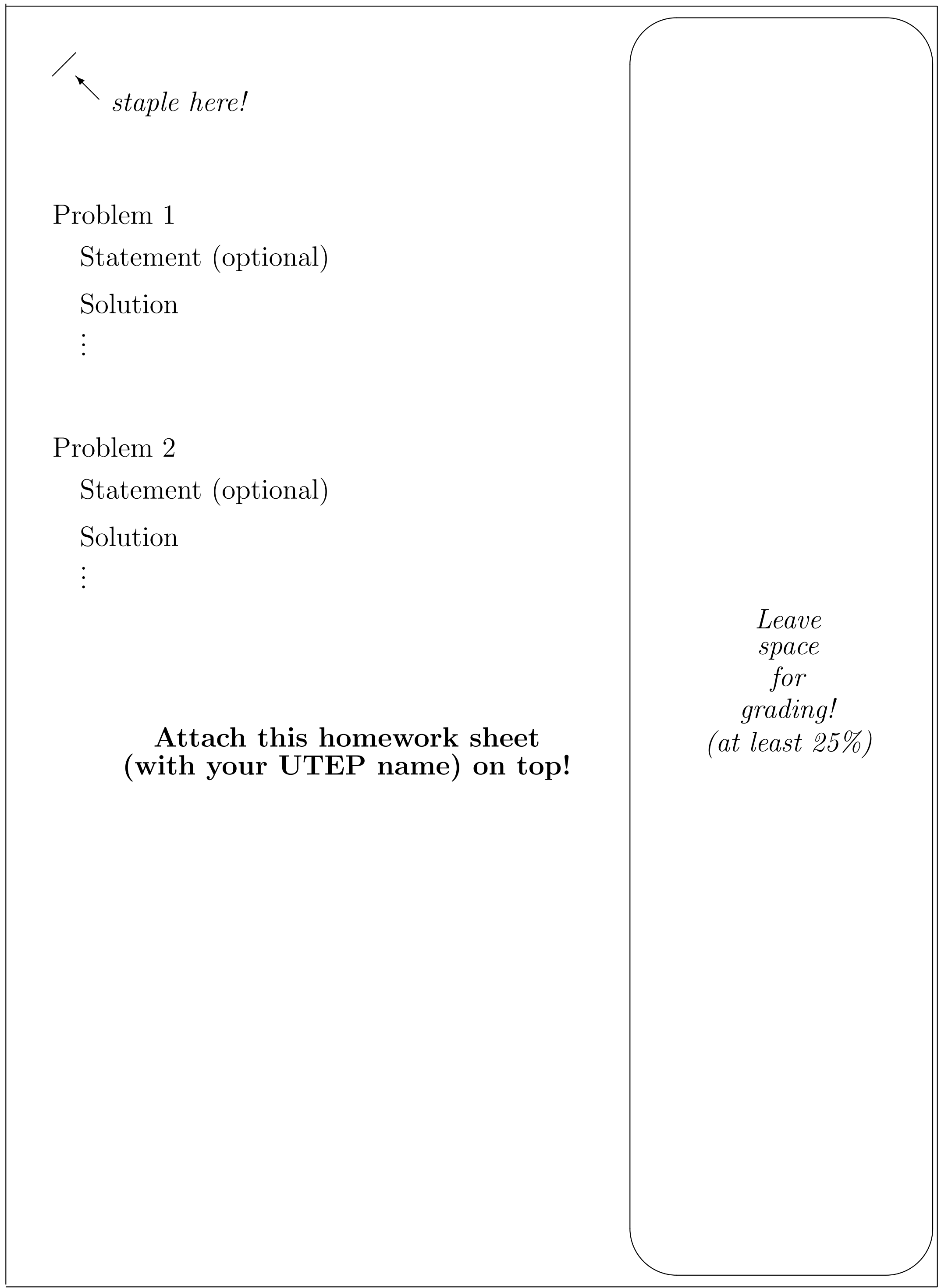

Figure 1: Homework Layout