Math 3341 **Homework 2** **Fall 2011**

The assignment is due at the beginning of class on September 19, 2011.

Problem 1 (10 points) Let $I_n = [a_n, b_n]$ be a nested sequence of closed bounded intervals. Suppose that $\lim_{n\to\infty}(b_n - a_n) = 0$. Show that $\bigcap_{n=1}^{\infty} I_n$ consists of exactly one point.

Problem 2 (10 points) Show that each real number in $[0, 1]$ has a unique decimal representation **except** when its decimal representation terminates. In this case, show that the number has exactly two decimal representations. (Example: $0.5 = 0.4\overline{9}$.)

What can you say about the cardinality of real numbers with two decimal expansions?

Problem 3 (10 points) Show that $[0, 1]$ has the same cardinality as $[0, 1] \times [0, 1]$.

Problem 4 (10 points) Show that $[0, 1]$ has the same cardinality as $(0, 1)$.

Problem 5 (10 points) Using the definition of limit, show that the sequence $(a_n)_{n=1}^{\infty}$, given by

$$a_n = \left(\frac{2n+5}{n+1}\right)^2,$$

converges to 4.