Math 3341 **Homework 4** **Fall 2011**

The assignment is due at the beginning of class on October 12, 2011.

Problem 1 (10 points) Let $(a_n)_{n=1}^{\infty}$ be a Cauchy sequence, and let $\varphi : \mathbb{N} \to \mathbb{N}$ be a one-to-one function. Show that the sequence $(a_{\varphi(n)})_{n=1}^{\infty}$ is a Cauchy sequence.

Problem 2 (10 points) Suppose (a_n) is a Cauchy sequence, and that (b_n) is a sequence satisfying $\lim_{n\to\infty} |a_n - b_n| = 0$. Show that (b_n) is a Cauchy sequence.

Problem 3 (10 points) Let us denote the set of all Cauchy sequences by $\mathcal{C}$.

Given two Cauchy sequences (a_n) and (b_n), we say $(a_n) \preceq (b_n)$ if for all $k \in \mathbb{N}$ there is an $N \in \mathbb{N}$ such that $a_n \leq b_n + \frac{1}{k}$ for all $n \geq N$.

1. Show that $\preceq$ is transitive on $\mathcal{C}$.
2. Show for all $(a_n), (b_n) \in \mathcal{C}$: If $(a_n) \preceq (b_n)$ and $(b_n) \preceq (a_n)$, then $\lim_{n\to\infty} |a_n - b_n| = 0$.

Problem 4 (10 points) Exercise 2.7.6.

Problem 5 (10 points) Consider the following two properties:

(1) Every non-empty set that is bounded from above has a supremum.

(2) Every Cauchy sequence converges.

Show that (2)$\Rightarrow$(1).