Math 3341   **Homework 6**   **Fall 2011**

The assignment is due at the beginning of class on November 7.

For this problem set, *open interval* means one of the following: $(-\infty, a)$, (a, b), (a, ∞), $(-\infty, \infty)$ with $a < b$, while *closed interval* refers to one of the following: $(-\infty, a]$, $[a, b]$, $[a, \infty)$ with $a \leq b$.

Problem 1 (10 points) Show that every open interval is the union of a countable collection of compact sets.

Problem 2 (10 points) Show that every non-empty open set is the union of a finite or countable collection of pairwise disjoint open intervals.

Problem 3 (10 points) True or false: Every non-empty closed set is the union of a finite or countable collection of closed intervals.

Problem 4 (10 points)   1. Show that every open set is the union of a countable collection of closed sets.

     2. Show that every closed set is the intersection of a countable collection of open sets.

Problem 5 (10 points) Show that $\mathbb{R} \setminus \mathbb{Q}$ is the intersection of a countable collection of open sets.