Analysis of the US Population Data

(see Worksheet 1)

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The Census Data

```
pop = {3.9, 5.3, 7.2, 9.6, 12.9, 17.1, 23.1, 31.4, 38.6, 50.2, 62.9,
   76.0, 92.0, 105.7, 122.8, 131.7, 150.7, 179.0, 205.0, 226.5, 248.7};
```

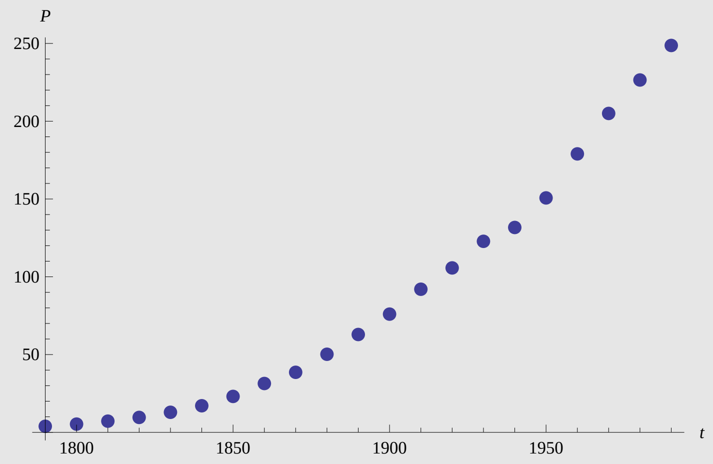
Here are the data from the US Census from 1790 to 1990. The first coordinate contains the year, the second the population (in millions):

```
data = Transpose[{Table[k, {k, 1790, 1990, 10}], pop}]
```

1790	3.9
1800	5.3
1810	7.2
1820	9.6
1830	12.9
1840	17.1
1850	23.1
1860	31.4
1870	38.6
1880	50.2
1890	62.9
1900	76.
1910	92.
1920	105.7
1930	122.8
1940	131.7
1950	150.7
1960	179.
1970	205.
1980	226.5
1990	248.7

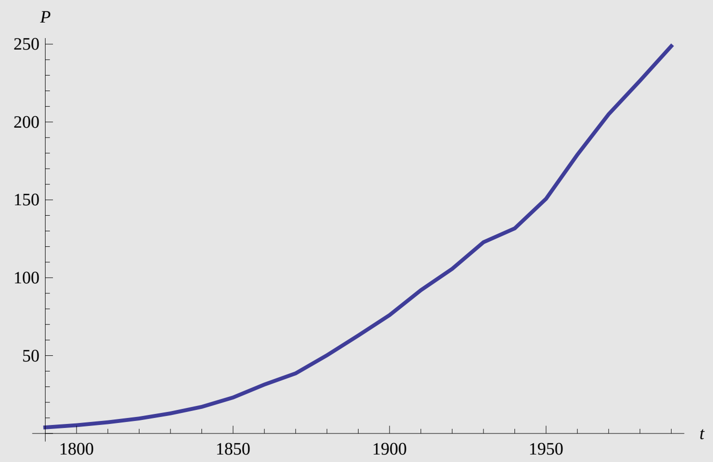
The graph of the function P=f(t), where t is time (measured in Year), P is population (in million people):

```
ListPlot[data, PlotStyle → AbsolutePointSize[7], AxesLabel → {t, P}]
```



"Connecting the dots":

```
ListPlot[data, PlotStyle → AbsoluteThickness[2], Joined → True, AxesLabel → {t, P}]
```



You can see the impact of the Great Depression in the 1930 s.

Rates of Change

Rate of change of the population in 1900, using the Census data from 1890 and 1900:

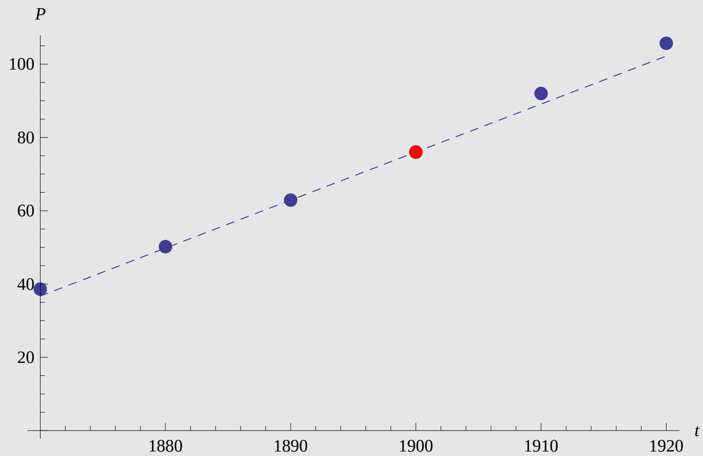
```
(76.0 - 62.9) / 10
```

1.31

"In 1900 the population grew at an approximate rate of 1.31 million per year."

The graph with the approximate tangent line (dashed) at t = 1900:

```
Show[ListPlot[data[[9 ;; 14]], PlotStyle → AbsolutePointSize[7], AxesLabel → {t, P}],
 Plot[76 + 1.31 (x - 1900), {x, 1870, 1920}, PlotStyle → Dashed],
 Epilog → {Red, AbsolutePointSize[7], Point[data[[12]]]}]
```



Zooming in sufficiently close, every differentiable function resembles a line.

Rate of change of the population in 1945, using the closest census date from 1940 and 1950:

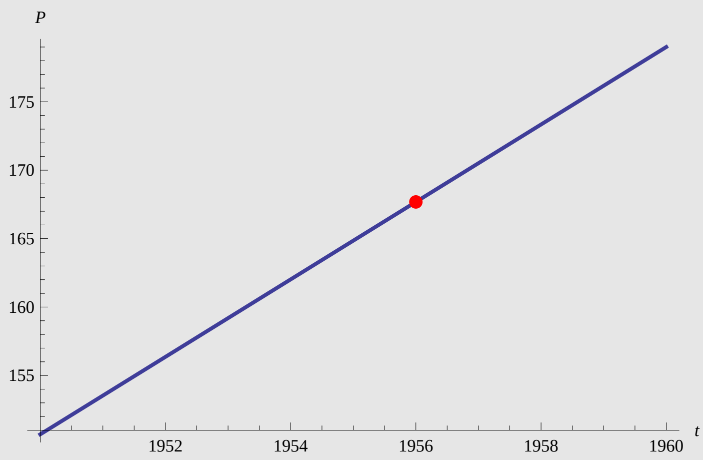
```
(150.7 - 131.7) / 10
```

1.9

"In 1945 the population grew at an approximate rate of 1.9 million per year."

Estimate for the population in 1956:
We need to (will) assume that the graph is **linear** between 1950 and 1960.

```
ListPlot[data[[17 ;; 18]], PlotStyle → AbsoluteThickness[2], Joined → True,
 Epilog → {AbsolutePointSize[7], Red, Point[{1956, 150.7 + 0.6 (179 - 150.7)}]},
 AxesLabel → {t, P}]
```



For the desired approximation we will use the equation of the line passing through the dataset points for t = 1950 and t = 1960.

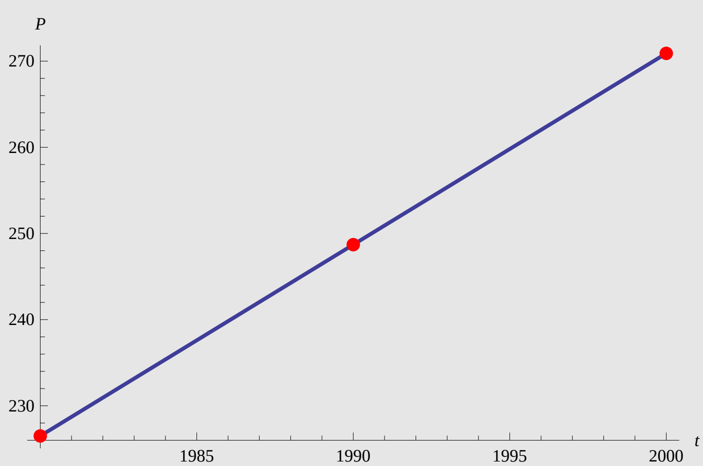
```
150.7 + 0.6 (179 - 150.7)
```

167.68

Estimate for the population in 2000:
We will (need to) assume that the graph is **linear** between 1980 and 2000.

```
data2 = Append[data, {2000, 248.7 + (248.7 - 226.5)}];
```

```
ListPlot[data2[[20 ;; 22]], PlotStyle → AbsoluteThickness[2], Joined → True,
 Epilog → {AbsolutePointSize[7], Red, Point[data2[[20 ;; 22]]]}, AxesLabel → {t, P}]
```



```
248.7 + (248.7 - 226.5)
```

270.9

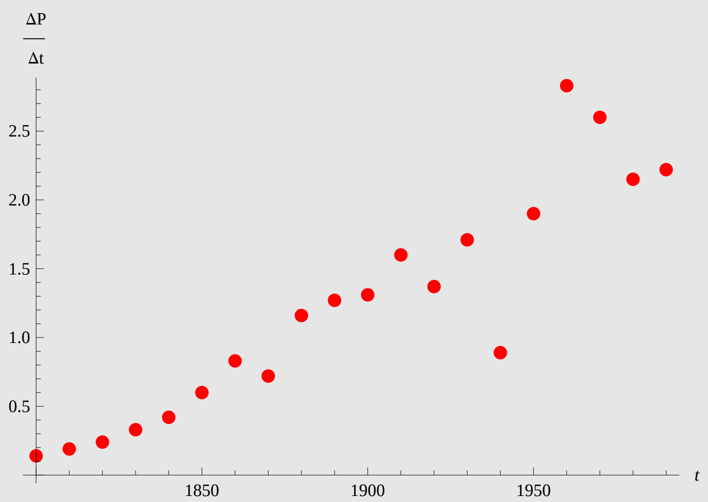
The actual population in the year 2000 was 281.4 million, so the method above turns out to undercount the actual population by more than 10 million. Can you come up wth a better idea which may have prevented the undercount?

The 2010 Census lists the US population at 308.7 million.

When was the rate of change of the population the greatest?

Below is the graph of the rate of change of population over time:

```
popd = Drop[pop - RotateRight[pop], 1] / 10;
ListPlot[Transpose[{Table[k, {k, 1800, 1990, 10}], popd}],
 PlotStyle → {Red, AbsolutePointSize[7]}, AxesLabel → {t, ΔP/Δt}]
```

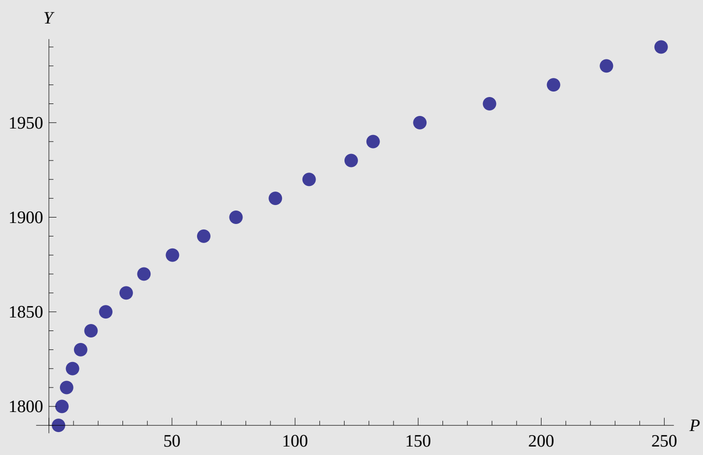


The rate of change was greatest during the period from 1950 to 1960, the "Baby Boom" era.
During that time the population grew at a rate of 2.83 million per year.

The Inverse Function

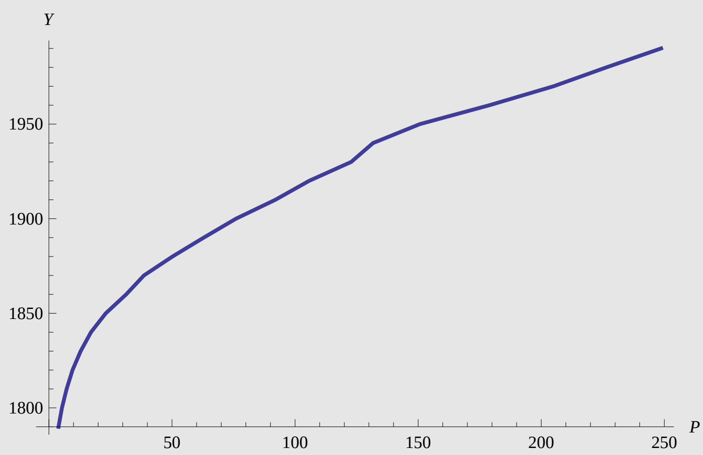
The graph of the inverse function $Y = f^{-1}(P)$, where P is population (in million people) and Y is time (measured in year):

```
datinv = Transpose[{pop, Table[k, {k, 1790, 1990, 10}]}];
ListPlot[datinv, PlotStyle → AbsolutePointSize[7], AxesLabel → {P, Y}]
```



"Connecting the dots":

```
ListPlot[datinv, PlotStyle → AbsoluteThickness[2], Joined → True, AxesLabel → {P, Y}]
```



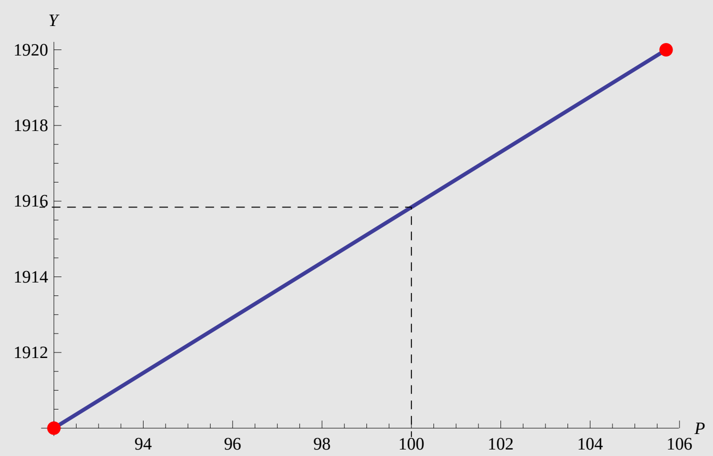
Estimate for $f^{-1}(100)$:
Below are the closest data points:

```
datinv[[13 ;; 14]]
```

$$\begin{pmatrix} 92. & 1910 \\ 105.7 & 1920 \end{pmatrix}$$

Using a linear interpolation we obtain the estimate $f^{-1}(100) =$ 1915.84.

```
ListPlot[datinv[[13 ;; 14]], PlotStyle → AbsoluteThickness[2], Joined → True,
 Epilog → {{Dashed, Line[{{100, 0}, {100, 1915.84}, {0, 1915.84}}]},
   {Red, AbsolutePointSize[7], Point[datinv[[13 ;; 14]]]}}, AxesLabel → {P, Y}]
```



“The US population reached 100 million in 1915”.

Estimate for $\left(f^{-1}\right)'(100)$, using the same data points:

```
(1920 - 1910) / (105.7 - 92)
```

0.729927

“When the US population was 100 million, it took about 0.73 years for the US popultion to grow by one million.”

Reference

http : // en.wikipedia.org/wiki/Demographics_of _the _United _States