## US Population

Census figures for the US population (in millions) are listed in Table 2.17. Let $f$ be the function such that $P=f(t)$ is the population (in millions) in year $t$.

TABLE 2.17 US population (in millions), 1790-1990

| Year | Population | Year | Population | Year | Population | Year | Population |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1790 | 3.9 | 1850 | 23.1 | 1910 | 92.0 | 1970 | 205.0 |
| 1800 | 5.3 | 1860 | 31.4 | 1920 | 105.7 | 1980 | 226.5 |
| 1810 | 7.2 | 1870 | 38.6 | 1930 | 122.8 | 1990 | 248.7 |
| 1820 | 9.6 | 1880 | 50.2 | 1940 | 131.7 |  |  |
| 1830 | 12.9 | 1890 | 62.9 | 1950 | 150.7 |  |  |
| 1840 | 17.1 | 1900 | 76.0 | 1960 | 179.0 |  |  |

(a) (i) Estimate the rate of change of the population for the years 1900, 1945, and 1990.
(ii) When, approximately, was the rate of change of the population greatest?
(iii) Estimate the US population in 1956.
(iv) Based on the data from the table, what would you predict for the census in the year 2000?
(b) Assume that $f$ is increasing (as the values in the table suggest). Then $f$ is invertible.
(i) What is the meaning of $f^{-1}(100)$ ?
(ii) What does the derivative of $f^{-1}(P)$ at $P=100$ represent? What are its units?
(iii) Estimate $f^{-1}(100)$.
(iv) Estimate the derivative of $f^{-1}(P)$ at $P=100$.

