Math 3341 **Homework 1** **Spring 2012**

The assignment is due at the beginning of class on January 31, 2012.

Problem 1 We say that m is the *maximum* of a set A if $m \in A$ and $m \geq a$ for all $a \in A$.

Suppose a set A of real numbers has a maximum, call it m. Show that m is also the supremum of A.

Problem 2 Let $A = \{x \in \mathbb{Q} \mid x^2 \leq 3\}$. Show that A is bounded from above, but that A has no maximum.

Problem 3 Let A and B be two sets of real numbers.

1. Assume that $\emptyset \neq A \subseteq B$ and that B is bounded from above. Show that A is bounded from above, and moreover that $\sup A \leq \sup B$.

2. State a similar theorem for sets that are bounded from below. (No proof required.)

Problem 4 A set is called *bounded* if it is bounded from above and bounded from below.

Assume that every non-empty bounded set of real numbers has both an infimum and a supremum. Show that this implies the Completeness Axiom for the Real Numbers.
Hint: This is not as easy as it may look at first glance.

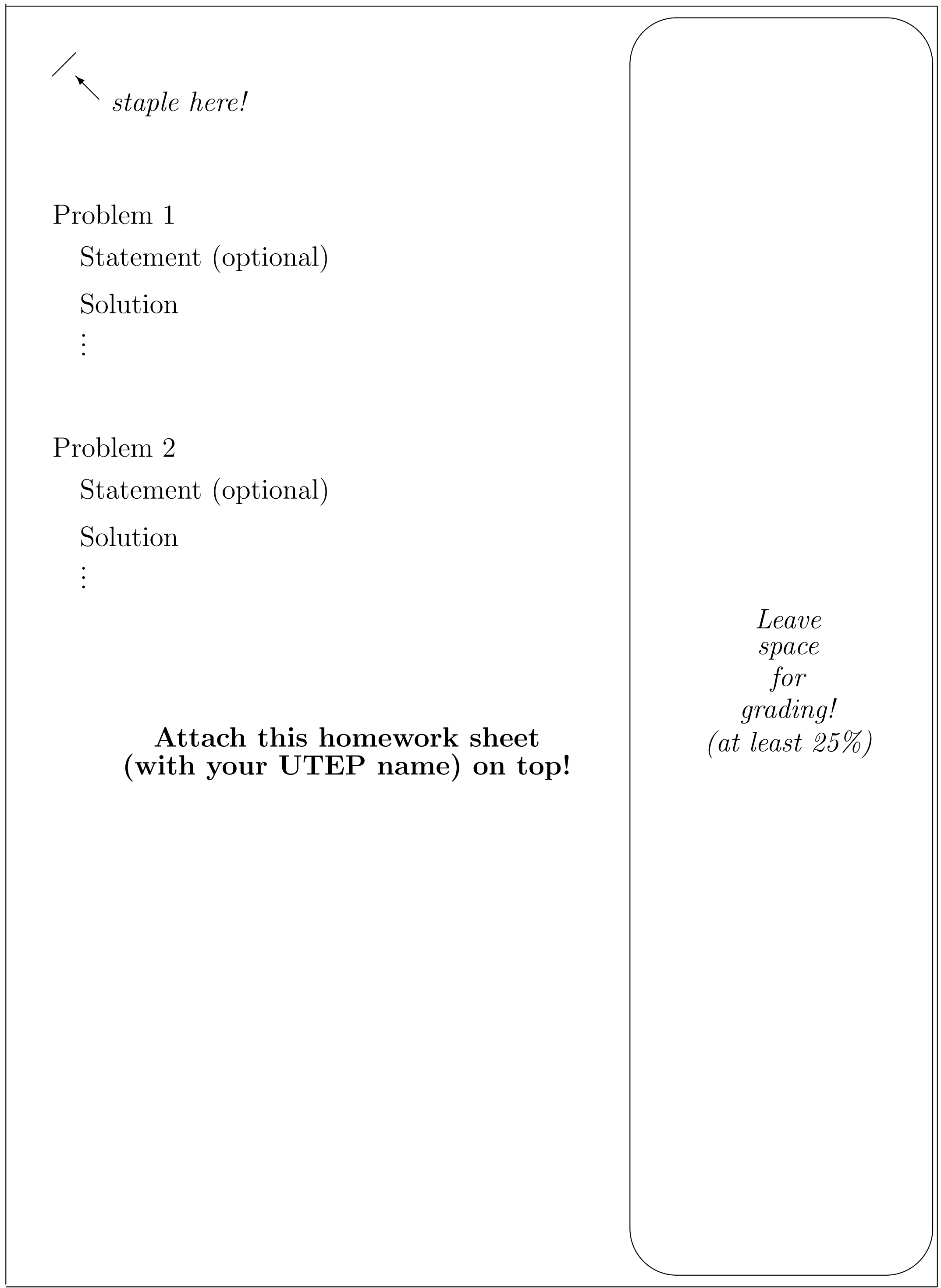

Figure 1: Homework Layout