Math 3341 **Homework 2** **Spring 2012**

The assignment is due at the beginning of class on February 9, 2012.

Problem 1 (10 points)

1. Show that each real number in $[0, 1]$ has a unique decimal representation **except** when its decimal representation terminates. In this case, show that the number has exactly two decimal representations. (Example: $0.51 = 0.50\overline{9}$.)
2. Show that the set of real numbers in $[0, 1]$ with two decimal expansions is countable.

Problem 2 (10 points)

1. Show that $(0, 1)$ has the same cardinality as $\mathbb{R}$.
2. Show that $[0, 1]$ has the same cardinality as $(0, 1)$.

Problem 3 (10 points) Using the definition of limit, show that the sequence $(a_n)_{n=1}^{\infty}$, given by

$$a_n = \sqrt{\frac{2n+5}{n+1}},$$

converges to $\sqrt{2}$.

Problem 4 (10 points) Show that limits are unique: Suppose that the sequence (a_n) converges to both a and b. Show that $a = b$.

Problem 5 (10 points) Exercise 2.3.11.