**Math 3341** **Homework 4** **Spring 2012**

***The assignment is due at the beginning of class on March 6, 2012.***

For Problems 1–3 do not use the fact that Cauchy sequences are convergent sequences.

**Problem 1 (10 points)** Suppose $(a_n)$ is a Cauchy sequence, and that $(b_n)$ is a sequence satisfying $\lim_{n\to\infty} |a_n - b_n| = 0$. Show that $(b_n)$ is a Cauchy sequence.

**Problem 2 (10 points)** Let $(a_n)_{n=1}^{\infty}$ be a Cauchy sequence, and let $\varphi : \mathbb{N} \to \mathbb{N}$ be a one-to-one function. Show that the sequence $(a_{\varphi(n)})_{n=1}^{\infty}$ is a Cauchy sequence.

**Problem 3 (10 points)** A Cauchy sequence $(a_n)$ is said to be *positive*, if for all $k \in \mathbb{N}$ there is an $N \in \mathbb{N}$ such that $a_n > -\frac{1}{k}$ for all $n \geq N$.

1. Show that the sum of two positive Cauchy sequences is positive.
2. Show that the product of two positive Cauchy sequences is positive.

**Problem 4 (10 points)** Exercise 2.7.6.

**Problem 5 (10 points)** Consider the following two properties:

(1) Every non-empty set that is bounded from above has a supremum.

(2) Every Cauchy sequence converges.

Show that (2)$\Rightarrow$(1).