Math 3341 **Homework 7** **Spring 2012**

The assignment is due at the beginning of class on April 27.

Problem 1 (10 points) Let $f : [a, b] \to \mathbb{R}$ be an increasing function. Show that $\lim_{x \to a} f(x)$ exists. What can you say about the relationship between this limit and $f(a)$?

Problem 2 (10 points) Let $f, g : \mathbb{R} \to \mathbb{R}$ be two continuous functions. Define

$$h(x) = \max\{f(x), g(x)\} \text{ for all } x \in \mathbb{R}.$$

Show that h is continuous on $\mathbb{R}$.

Problem 3 (10 points) Let $f : \mathbb{R} \to \mathbb{R}$ be continuous on $\mathbb{R}$, and assume that for all $\varepsilon > 0$ there is an $N > 0$ such that $|f(x)| < \varepsilon$ for all x satisfying $|x| > N$. Show that f is uniformly continuous on $\mathbb{R}$.

Let $f : [a, b] \to \mathbb{R}$ be a function. We say f satisfies $(*)$ if there is an $M > 0$ such that $|f(x) - f(y)| \le M \cdot |x - y|$ for all $x, y \in [a, b]$.

Problem 4 (10 points) Show: If f satisfies $(*)$, then f is uniformly continuous on $[a, b]$.

Problem 5 (10 points) Let $f : [0, 1] \to \mathbb{R}$ be given by $f(x) = \sqrt{x}$. Show that f does not satisfy $(*)$.