Worksheet #4

- 1. Graph the function $f : \mathbb{R} \to \mathbb{R}$, given by $f(x) = x^3 3x$.
- 2. Let R be an equivalence relation on a non-empty set A, and let $a \in A$. Give the definition for [a], the equivalence class of a under R.
- 3. Now consider the following equivalence relation R on \mathbb{R} :

$$xRy \quad \Leftrightarrow \quad x^3 - 3x = y^3 - 3y.$$

- (a) Find the equivalence class [5].
- (b) Find the equivalence class [0].
- (c) Find all $x \in \mathbb{R}$ such that [x] has exactly 2 elements.
- 4. Define a new function g on the equivalence classes given by the equivalence relation R by setting g([x]) = f(x).
 - (a) Show that the function g is well-defined, i.e. show the following:

If
$$[x_1] = [x_2]$$
, then $g([x_1]) = g([x_2])$.

(b) Show that g is injective.