Math 2325 **A Strange Sum ...** **August 27, 2012**

For non-negative reduced fractions, define their "sum" as follows:

$$\frac{a}{b} \oplus \frac{c}{d} = \frac{a+c}{b+d}$$

1. Compute a few examples.
2. Draw some pairs of reduced fractions and their "sum" on the real number line. Can you make a conjecture regarding the relative position of the three numbers? Can you prove your conjecture?
3. Set $F_0 = \{\frac{0}{1}, \frac{1}{1}\}$. Then insert their "sum" (and sort by size) to define $F_1 = \{\frac{0}{1}, \frac{1}{2}, \frac{1}{1}\}$. F_2 is obtained by inserting all "sums" of neighbors in F_1:
$$F_2 = \{\frac{0}{1}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{1}{1}\}$$
Compute F_n for $n = 3, \ldots, 7$.
4. Will the number of elements in F_n grow like a polynomial or like an exponential function? Can you find an argument to support your conjecture? Can you come up with a general formula for the number of elements in F_n?
5. Will one eventually obtain all rational numbers between 0 and 1 in this fashion? Give an argument why this is true, or find a counterexample.