

The assignment is due at the beginning of class on February 11, 2013.

Problem 1 (15 points) You have seen how to generate compound statements using the four connectives \neg , \vee , \wedge and \Rightarrow . This problem addresses the question whether all four connectives are necessary.

- Use a truth table to show that $A \Rightarrow B$ is equivalent to $\neg(A \wedge \neg B)$.
- Show that $A \vee B$ can be written using only the connectives \neg and \wedge .

Thus the two connectives \neg and \wedge suffice to generate all compound statements. It is possible to further reduce to only one connective, albeit a different one: Let us define the new connective NOR by setting

$$A \text{ NOR } B \iff \neg(A \vee B).$$

- Show that the four compound statements $\neg A$, $A \vee B$, $A \wedge B$ and $A \Rightarrow B$ can be written using only the NOR-connective.

Problem 2 (10 points) Negate¹ the following statement: “All dogs have three legs, or there is a cat with two tails.”

Problem 3 (15 points) In each case, give an example, or explain why such an example cannot exist:

- Is there a predicate $A(x, y)$ such that the statement $\forall x \exists y : A(x, y)$ is true, while the statement $\exists y \forall x : A(x, y)$ is false?
- Is there a predicate $A(x, y)$ such that the statement $\exists y \forall x : A(x, y)$ is true, while the statement $\forall x \exists y : A(x, y)$ is false?

Problem 4 (10 points) A clothing store advertises: “For every customer we have a rack of clothes that fit.”

- Write the statement above using quantifier(s) and predicate(s).
- Negate the sentence using quantifier(s) and predicate(s).
- Write the negation¹ in the form of an English sentence.

¹Don't just write “*It is not true that ...*”

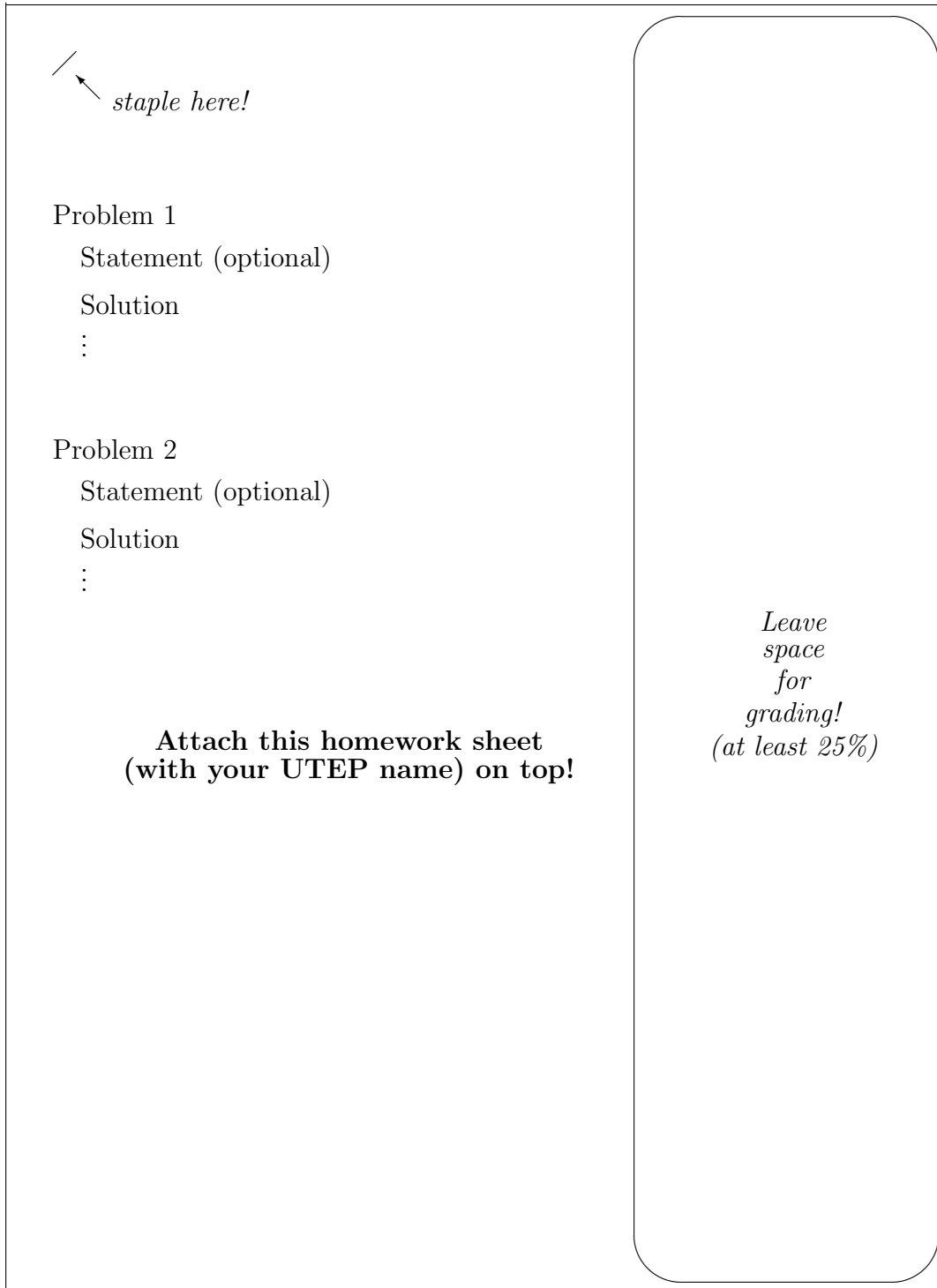


Figure 1: Homework Layout