Math 3325 **Homework 5** **Spring 2013**

The assignment is due at the beginning of class on April 8, 2013.

Problem 1 (10 points) Using the *Well-ordering Principle*, show that every polynomial (with real coefficients) can be written as a product of irreducible polynomials.

Problem 2 (10 points)

1. Show that every positive integer can be written as the sum of (one or more) distinct powers of 2. (Examples: $8 = 2^3$, $25 = 2^4 + 2^3 + 2^0$.)

2. Can every positive integer be written as the sum of (one or more) distinct powers of 3?

Problem 3 (10 points) Use induction to show the following theorem: If a set has n elements, then its power set has 2^n elements. (See Homework 3 Problem 3.)

For a natural number n, let $\mathcal{D}_n$ denote the set of the divisors of n. For example, $\mathcal{D}_{42} = \{1, 2, 3, 6, 7, 14, 21, 42\}$ and $\mathcal{D}_{12} = \{1, 2, 3, 4, 6, 12\}$. For $m, n \in \mathbb{N}$ let $m \sqcap n$ denote the greatest common divisor of n and m, and $m \sqcup n$ their least common multiple. For instance $6 \sqcap 4 = 2$ and $6 \sqcup 4 = 12$. It turns out that $\mathcal{D}_{42}$ with these two operations $\sqcap$ and $\sqcup$ forms a Boolean Algebra, while $\mathcal{D}_{12}$ does **not**.

Problem 4 (10 points) Verify Boolean Algebra Laws 5, 6 and 7 for $\mathcal{D}_{42}$.

Problem 5 (10 points)

1. Show that $\mathcal{D}_{12}$ does not form a Boolean Algebra.

2. Conjecture for which values of n the set $\mathcal{D}_n$ forms a Boolean Algebra.