

*The assignment is due at the beginning of class on April 22, 2013.*

**Problem 1 (10 points)** Let  $A \subseteq \mathbb{R}$ . An element  $m \in A$  is called *maximum* of  $A$ , if  $m \geq a$  for all  $a \in A$ . Show that every finite non-empty set of real numbers has a maximum.

**Problem 2 (10 points)** Let  $R$  be a relation from the set  $A$  to the set  $B$ , and  $S$  be a relation from the set  $B$  to the set  $C$ . Prove that  $(S \circ R)^{-1} = R^{-1} \circ S^{-1}$ .

**Problem 3 (10 points)** Let  $R$  be a relation from  $A$  to  $B$ . For an element  $b \in B$  define the set  $R_b := \{a \in A \mid (a, b) \in R\}$ . Show

$$\bigcup_{b \in B} R_b = \text{Dom } R.$$

**Problem 4 (10 points)** Define a relation  $R$  on  $\mathbb{R}$  as follows:  $a R b$  if  $a - b$  is irrational. Prove or disprove:  $R$  is (a) reflexive, (b) symmetric, (c) transitive.

**Problem 5 (10 points)** Let  $S = \{(x, y) \mid x^2 + (y - 1)^2 = 4\}$  and  $T = \{(x, y) \mid x^2 + y^2 \leq 1\}$  be two relations on  $\mathbb{R}$ . Find  $T \circ S$ .