# RPN Calculators and Mathematics Competitions 

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- RPN was proposed in the 1950s by Australian philosopher Charles L Hamblin.
- The RPN calculator was invented using Reverse Polish Notation and was introduced in the late 1960s and early 1970s.


## Example of Algebraic

$$
((15 /(7-(1+1))) * 3)-(2+(1+1))=
$$

## Example in Polish Notation

$$
-* / 15-7+113+2+11
$$

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$$
\begin{array}{lllllllllllllll}
- & * & / & 15 & - & 7 & + & 1 & 1 & 3 & + & 2 & + & 1 & 1 \\
- & * & / & 15 & - & 7 & & 2 & & 3 & + & 2 & + & 1 & 1 \\
- & * & / & 15 & & & 5 & & & 3 & + & 2 & + & 1 & 1
\end{array}
$$

## Example in Polish Notation

\[

\]

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| - | $*$ | $/$ | 15 | - | 7 | + | 1 | 1 | 3 | + | 2 | + | 1 | 1 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| - | $*$ | $/$ | 15 | - | 7 |  | 2 |  | 3 | + | 2 | + | 1 | 1 |
| - | $*$ | $/$ | 15 |  |  | 5 |  |  | 3 | + | 2 | + | 1 | 1 |
| - | $*$ |  |  | 3 |  |  |  | 3 | + | 2 | + | 1 | 1 |  |
| - |  |  |  | 9 |  |  |  |  |  |  | 2 | + | 1 | 1 |

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| - | * | 15 | - | 7 | + | 1 | 1 | 3 | + | 2 | + | 1 | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| - | * | 15 | - | 7 |  | 2 |  | 3 | + | 2 | + | 1 | 1 |
| - | * |  |  |  | 5 |  |  | 3 | + | 2 | + | 1 | 1 |
| - | * |  |  | 3 |  |  |  | 3 | + | 2 | + | 1 | 1 |
| - |  |  |  | 9 |  |  |  |  | $+$ | 2 | + | 1 | 1 |
| - |  |  |  | 9 |  |  |  |  | $+$ | 2 |  | 2 |  |

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| - |  |  |  | 9 |  |  |  |  | + | 2 | + | 1 | 1 |  |
| - |  |  |  | 9 |  |  |  |  | + | 2 |  | 2 |  |  |
| - |  |  |  | 9 |  |  |  |  |  | 4 |  |  |  |  |

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| - |  |  |  | 9 |  |  |  |  | + | 2 | + | 1 | 1 |  |
| - |  |  |  | 9 |  |  |  | + | 2 |  | 2 |  |  |  |
| - |  |  |  | 9 |  |  |  |  |  |  | 4 |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

## Polish v. Reverse Polish

Polish/Prefix:

$$
-* / 15-7+113+2+11
$$

RPN/Postfix:

$$
15711+-/ 3 * 211++-
$$

## Example in Reverse Polish Notation



## Example in Reverse Polish Notation

$\begin{array}{llllllllllllllll}15 & 7 & 1 & 1 & + & - & / & 3 & * & 2 & 1 & 1 & + & + & - \\ 15 & 7 & & 2 & & - & / & 3 & * & 2 & 1 & 1 & + & + & -\end{array}$

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| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 15 | 7 |  | 2 |  | - | $/$ | 3 | $*$ | 2 | 1 | 1 | + | + | - |
| 15 |  |  | 5 |  | $/$ | 3 | $*$ | 2 | 1 | 1 | + | + | - |  |
|  |  |  | 3 |  |  | 3 | $*$ | 2 | 1 | 1 | + | + | - |  |

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| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 15 | 7 |  | 2 |  | - | $/$ | 3 | $*$ | 2 | 1 | 1 | + | + | - |
| 15 |  |  | 5 |  |  | $/$ | 3 | $*$ | 2 | 1 | 1 | + | + | - |
|  |  |  | 3 |  |  |  | 3 | $*$ | 2 | 1 | 1 | + | + | - |
|  |  |  |  | 9 |  |  |  |  | 2 | 1 | 1 | + | + | - |
|  |  |  |  | 9 |  |  |  |  | 2 |  | 2 |  | + | - |
|  |  |  |  | 9 |  |  |  |  |  |  | 4 |  |  | - |
|  |  |  |  |  |  | 5 |  |  |  |  |  |  |  |  |

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- The processes used improve student understanding of mathematics
- The RPN allows users to see intermediate results
- In most cases the RPN saves keystrokes and thus saves time
- Most Math Competitions are won by competitors who use RPN Calculators


## Special Guest

## Mike Harmon, YISD

RPNs in high school math competitions

## QUESTIONS?

