Math 4303 **Worksheet #4**

1. Graph the function $f : \mathbb{R} \to \mathbb{R}$, given by $f(x) = x^3 - 3x$.

2. Let R be an equivalence relation on a non-empty set A, and let $a \in A$. Give the definition for $[a]$, the equivalence class of a under R.

3. Now consider the following equivalence relation R on $\mathbb{R}$:

$$xRy \quad \Leftrightarrow \quad x^3 - 3x = y^3 - 3y.$$

   (a) Find the equivalence class $[5]$.

   (b) Find the equivalence class $[0]$.

   (c) Find all $x \in \mathbb{R}$ such that $[x]$ has exactly 2 elements.

4. Define a new function g on the equivalence classes given by the equivalence relation R by setting $g([x]) = f(x)$.

   (a) Show that the function g is well-defined, i.e. show the following:

$$\text{If } [x_1] = [x_2], \text{ then } g([x_1]) = g([x_2]).$$

   (b) Show that g is injective.