**Math 3335** **Formula Sheet for Test 2** **Fall 2015**

**Integrals.** Let $\vec{\boldsymbol{r}}(t) = (x(t), y(t), z(t)) : [a, b] \to \mathbb{R}^3$ parametrize a curve $C$.

**Arclength.** $\int_C ds = \int_a^b \|\vec{\boldsymbol{r}}'(t)\|\, dt.$

**Line integral.** For a vector field $\vec{\boldsymbol{F}} = (F_1, F_2, F_3) : \mathbb{R}^3 \to \mathbb{R}^3$,

$$\int_C \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{r}} = \int_C F_1\, dx + F_2\, dy + F_3\, dz = \int_a^b \vec{\boldsymbol{F}}(\vec{\boldsymbol{r}}(t)) \cdot \vec{\boldsymbol{r}}'(t)\, dt.$$

**Path integral.** For a scalar-valued function $f : \mathbb{R}^3 \to \mathbb{R}$,

$$\int_C f\, ds = \int_a^b f(\vec{\boldsymbol{r}}(t))\|\vec{\boldsymbol{r}}'(t)\|\, dt.$$

**Integral Theorems in the Plane.** Let $\vec{\boldsymbol{F}} = (F_1, F_2) : \mathbb{R}^2 \to \mathbb{R}^2$ be a vector field, and let $R$ be a bounded domain in $\mathbb{R}^2$ with boundary curve $\partial R$, oriented in the standard way.

**Green's Theorem.**

$$\iint_R \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right) dx\, dy = \oint_{\partial R} F_1\, dx + F_2\, dy.$$

Adding 0 as a third component in $\vec{\boldsymbol{F}}$ and $\vec{\boldsymbol{r}}$, we can write this as

$$\iint_R (curl\, \vec{\boldsymbol{F}}) \cdot \vec{\boldsymbol{k}}\, dx\, dy = \oint_{\partial R} \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{r}}.$$

**Area Theorem.** In particular,

$$Area(R) = \iint_R dx\, dy = \frac{1}{2} \oint_{\partial R} x\, dy - y\, dx.$$

**Divergence Theorem.** Let $\vec{\boldsymbol{n}}$ denote the unit normal to $\partial R$, pointing outward. Then

$$\iint_R div\, \vec{\boldsymbol{F}}\, dx\, dy = \oint_{\partial R} \vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{n}}\, ds.$$