Bezier Curves

Math 3335

Helmut Knaust - Department of Mathematical Sciences, University of Texas at El Paso,
hknaust@utep.edu
9/15/16

2D - 1 control point

```
b0 = {1, 1}
b1 = {0, 2}
b2 = {2, 3}
```

$\{1, 1\}$

$\{0, 2\}$

$\{2, 3\}$

```
bc1[t_] = t b0 + (1 - t) b1
bc2[t_] = t b1 + (1 - t) b2
```

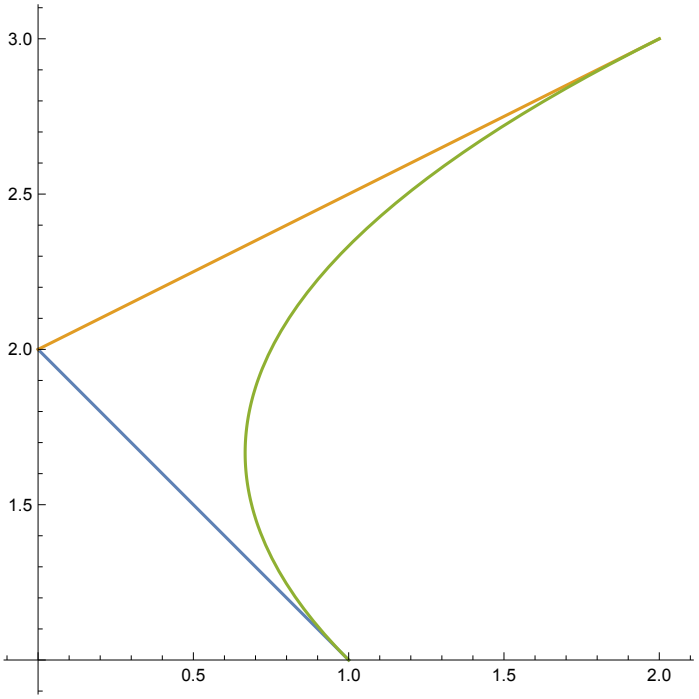
$\{t, 2\,(1-t)+t\}$

$\{2\,(1-t), 3\,(1-t)+2\,t\}$

```
bc[t_] = t bc1[t] + (1 - t) bc2[t]
```

$\left\{t^2+2\,(1-t)^2,\ t\,(2\,(1-t)+t)+(1-t)\,(3\,(1-t)+2\,t)\right\}$

```
ParametricPlot[{bc1[t], bc2[t], bc[t]}, {t, 0, 1}]
```



3D -1 control point

```
b0 = {1, 1, 2}
b1 = {0, 2, 1}
b2 = {2, 3, -1}
```

$\{1, 1, 2\}$

$\{0, 2, 1\}$

$\{2, 3, -1\}$

```
bc1[t_] = t b0 + (1 - t) b1
bc2[t_] = t b1 + (1 - t) b2
```

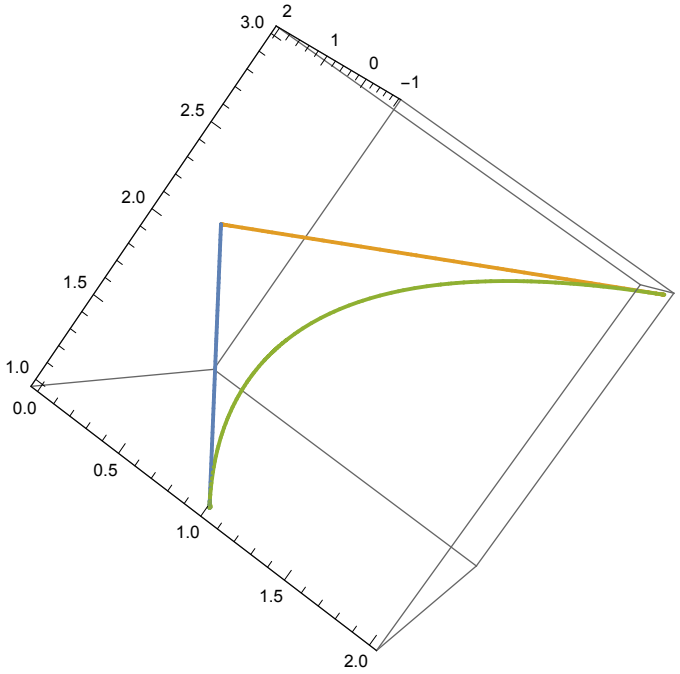
$\{t, 2\,(1-t)+t, t+1\}$

$\{2\,(1-t), 3\,(1-t)+2\,t, 2\,t-1\}$

```
bc[t_] = t bc1[t] + (1 - t) bc2[t]
```

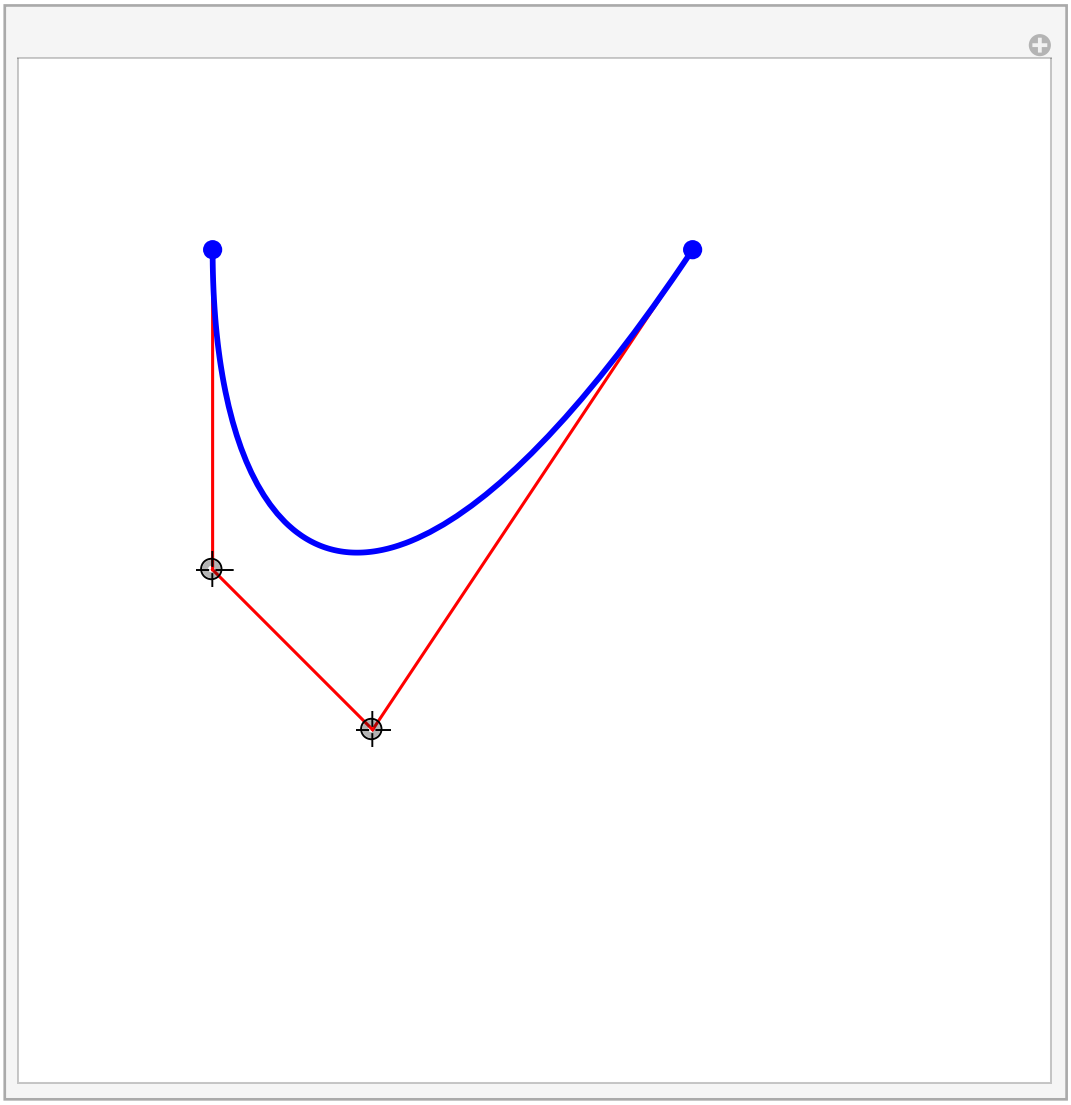
$\left\{t^2+2\,(1-t)^2, t\,(2\,(1-t)+t)+(1-t)\,(3\,(1-t)+2\,t), t\,(t+1)+(1-t)\,(2\,t-1)\right\}$

```
ParametricPlot3D[{bc1[t], bc2[t], bc[t]}, {t, 0, 1}]
```



2D - 2 control points

```
Manipulate[b0 = {1, 2};
 b3 = {-2, 2};
 bc1[t_] = t b0 + (1 - t) b1;
 bc2[t_] = t b1 + (1 - t) b2;
 bc3[t_] = t b2 + (1 - t) b3;
 c1[t_] = t bc1[t] + (1 - t) bc2[t];
 c2[t_] = t bc2[t] + (1 - t) bc3[t];
 bc[t_] = t c1[t] + (1 - t) c2[t];
 ParametricPlot[{bc1[t], bc2[t], bc3[t], bc[t]}, {t, 0, 1}, PlotStyle → {Red, Red,
    Red, {AbsoluteThickness[3], Blue}}, PlotRange → {{-3, 3}, {-3, 3}}, Axes → None,
  ImageSize → 500, Epilog → {Blue, AbsolutePointSize[10], Point[b0], Point[b3]}],
 {{b1, {-1, -1}}}, {{b2, {-2, 0}}, {-2, -2}, {2, 2}}, ControlType → Locator]
```



3D - 2 control points

```
b0 = {1, 1, 2}
b1 = {0, 2, 1}
b2 = {2, 3, 4}
b3 = {0, 1, 3}
```

$\{1, 1, 2\}$

$\{0, 2, 1\}$

$\{2, 3, 4\}$

$\{0, 1, 3\}$

```
bc1[t_] = t b0 + (1 - t) b1
bc2[t_] = t b1 + (1 - t) b2
bc3[t_] = t b2 + (1 - t) b3
```

$\{t, 2(1-t)+t, t+1\}$

$\{2(1-t), 3(1-t)+2t, 4(1-t)+t\}$

$\{2t, 2t+1, 3(1-t)+4t\}$

```
c1[t_] = t bc1[t] + (1 - t) bc2[t]
c2[t_] = t bc2[t] + (1 - t) bc3[t]
```

$\left\{t^2+2(1-t)^2,\ t(2(1-t)+t)+(1-t)(3(1-t)+2t),\ t(t+1)+(1-t)(4(1-t)+t)\right\}$

$\{4(1-t)t,\ (1-t)(2t+1)+t(3(1-t)+2t),\ t(4(1-t)+t)+(1-t)(3(1-t)+4t)\}$

```
bc[t_] = t c1[t] + (1 - t) c2[t]
```

$\left\{t\left(t^2+2(1-t)^2\right)+4t(1-t)^2,\ t(t(2(1-t)+t)+(1-t)(3(1-t)+2t))+(1-t)((1-t)(2t+1)+t(3(1-t)+2t)),\right.$
$\left.t(t(t+1)+(1-t)(4(1-t)+t))+(1-t)(t(4(1-t)+t)+(1-t)(3(1-t)+4t))\right\}$

```
Expand[%]
```

$\left\{7t^3-12t^2+6t,\ 3t^3-9t^2+6t+1,\ 8t^3-12t^2+3t+3\right\}$

```
ParametricPlot3D[{bc1[t], bc2[t], bc3[t], bc[t]}, {t, 0, 1},
 PlotStyle → {Red, Red, Red, {AbsoluteThickness[3], Blue}}]
```