Integrals. Let $\vec{\boldsymbol{r}}(t) = (x(t), y(t), z(t)) : [a, b] \to \mathbb{R}^3$ parametrize a curve C.

Arclength. $\int_C ds = \int_a^b \|\vec{\boldsymbol{r}}'(t)\|\, dt.$

Line integral with respect to arclength. For a scalar-valued function $f : \mathbb{R}^3 \to \mathbb{R}$,

$$\int_C f\, ds = \int_a^b f(\vec{\boldsymbol{r}}(t))\|\vec{\boldsymbol{r}}'(t)\|\, dt.$$

Line integral. For a vector field $\vec{\boldsymbol{F}} = (F_1, F_2, F_3) : \mathbb{R}^3 \to \mathbb{R}^3$,

$$\int_C \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{r}} = \int_C F_1\, dx + F_2\, dy + F_3\, dz = \int_a^b \vec{\boldsymbol{F}}(\vec{\boldsymbol{r}}(t)) \cdot \vec{\boldsymbol{r}}'(t)\, dt.$$

Integral Theorems in the Plane. Let $\vec{\boldsymbol{F}} = (F_1, F_2) : \mathbb{R}^2 \to \mathbb{R}^2$ be a vector field, that is smooth on a bounded region R in $\mathbb{R}^2$. Let C denote the boundary curve of R, oriented in the standard way.

Green's Theorem.

$$\iint_R \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right) dx\, dy = \oint_C F_1\, dx + F_2\, dy = \oint_C \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{r}}.$$

Area Theorem. In particular,

$$Area(R) = \iint_R dx\, dy = \oint_C x\, dy = -\oint_C y\, dx = \frac{1}{2} \oint_C x\, dy - y\, dx.$$