

Integrals. Let $\vec{r}(t) = (x(t), y(t), z(t)) : [a, b] \rightarrow \mathbb{R}^3$ parametrize a curve C .

Arclength. $\int_C ds = \int_a^b \|\vec{r}'(t)\| dt.$

Line integral with respect to arclength. For a scalar-valued function $f : \mathbb{R}^3 \rightarrow \mathbb{R}$,

$$\int_C f ds = \int_a^b f(\vec{r}(t)) \|\vec{r}'(t)\| dt.$$

Line integral. For a vector field $\vec{F} = (F_1, F_2, F_3) : \mathbb{R}^3 \rightarrow \mathbb{R}^3$,

$$\int_C \vec{F} \cdot d\vec{r} = \int_C F_1 dx + F_2 dy + F_3 dz = \int_a^b \vec{F}(\vec{r}(t)) \cdot \vec{r}'(t) dt.$$

Integral Theorems in the Plane. Let $\vec{F} = (F_1, F_2) : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a vector field, that is smooth on a bounded region R in \mathbb{R}^2 . Let C denote the boundary curve of R , oriented in the standard way.

Green's Theorem.

$$\iint_R \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right) dx dy = \oint_C F_1 dx + F_2 dy = \oint_C \vec{F} \cdot d\vec{r}.$$

Area Theorem. In particular,

$$\text{Area}(R) = \iint_R dx dy = \oint_C x dy = - \oint_C y dx = \frac{1}{2} \oint_C x dy - y dx.$$

More Integrals. Let R be a subset of \mathbb{R}^2 , and let $\vec{r}(u, v) = (x(u, v), y(u, v), z(u, v)) : R \rightarrow S$ parametrize a surface $S \subseteq \mathbb{R}^3$.

Surface area. $\iint_S dA = \iint_R \|\vec{r}_u \times \vec{r}_v\| du dv.$

Surface integral. For a scalar-valued function $f : S \rightarrow \mathbb{R}$,

$$\iint_S f dA = \iint_R f(\vec{r}(u, v)) \|\vec{r}_u \times \vec{r}_v\| du dv$$

Flux integral. Let $\vec{F} = (F_1, F_2, F_3) : S \rightarrow \mathbb{R}^3$ be a vector field, and let \vec{n} be a choice of unit normal for the orientable surface S . Then

$$\iint_S \vec{F} \cdot \vec{n} dA = \iint_R \vec{F}(\vec{r}(u, v)) \cdot (\vec{r}_u \times \vec{r}_v) du dv$$

Integral Theorems in \mathbb{R}^3 .

Gauss' Divergence Theorem. Let T be a closed bounded solid in \mathbb{R}^3 with boundary surface ∂T , oriented such that its unit normal points outward. Let $\vec{F} = (F_1, F_2, F_3) : T \rightarrow \mathbb{R}^3$ be a vector field.

$$\iiint_T \operatorname{div} \vec{F} \, dV = \iint_{\partial T} \vec{F} \cdot \vec{n} \, dA.$$

Stokes' Theorem. Let $S \subseteq \mathbb{R}^3$ be an orientable surface with the simple closed curve ∂S as its boundary. Let $\vec{F} = S \rightarrow \mathbb{R}^3$ be a vector field. Then

$$\iint_S (\operatorname{curl} \vec{F}) \cdot \vec{n} \, dA = \oint_{\partial S} \vec{F} \cdot d\vec{r}.$$

Here \vec{n} is a choice of unit normal for S , and ∂S is being traversed such that \vec{n} is to the left of the curve ∂S .