Math 2325 **A Strange Sum . . .** **Spring 2017**

For non-negative (reduced) fractions, define their "sum" as follows:

$$\frac{a}{b} \oplus \frac{c}{d} = \frac{a+c}{b+d}$$

1. Compute a few examples.

2. Draw some pairs of reduced fractions and their "sum" on the real number line. Can you make a conjecture regarding the relative position of the three numbers?

3. Set $F_0 = \left\{\frac{0}{1}, \frac{1}{1}\right\}$. Then insert their "sum" (and sort by size) to define $F_1 = \left\{\frac{0}{1}, \frac{1}{2}, \frac{1}{1}\right\}$. F_2 is obtained by inserting all "sums" of neighbors in F_1:

$$F_2 = \left\{\frac{0}{1}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{1}{1}\right\}$$

   Compute F_n for $n = 3, \ldots, 7$.

4. Will the number of elements in F_n grow like a polynomial or like an exponential function? Can you find an argument to support your conjecture? Can you come up with a general formula for the number of elements in F_n?

5. Will one eventually obtain all rational numbers between 0 and 1 in this fashion? Give an argument why this is true, or find a counterexample.

6. Can you prove your conjecture in 2.?