Integrals. Let $\vec{\boldsymbol{r}}(t) = (x(t), y(t), z(t)) : [a, b] \to \mathbb{R}^3$ parametrize a curve C.

Arclength. $\int_C ds = \int_a^b \|\vec{\boldsymbol{r}}'(t)\|\, dt = \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2}\, dt.$

Line integral with respect to arclength. For a scalar-valued function $f : \mathbb{R}^3 \to \mathbb{R}$,

$$\int_C f\, ds = \int_a^b f(\vec{\boldsymbol{r}}(t))\|\vec{\boldsymbol{r}}'(t)\|\, dt.$$

Line integral. For a vector field $\vec{\boldsymbol{F}} = (F_1, F_2, F_3) : \mathbb{R}^3 \to \mathbb{R}^3$,

$$\int_C \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{r}} = \int_C F_1\, dx + F_2\, dy + F_3\, dz = \int_a^b \vec{\boldsymbol{F}}(\vec{\boldsymbol{r}}(t)) \cdot \vec{\boldsymbol{r}}'(t)\, dt.$$