Questions for Algebra Teachers

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1. My teacher from last year told me that whatever I do to one side of an equation, I must do the same thing to the other side to keep the equality true. I can't figure out what I'm doing wrong by adding 1 to the numerator of both fractions in the equality $\frac{1}{2} = \frac{2}{4}$ and getting $\frac{2}{2} = \frac{3}{4}$.

2. Why does the book say that a polynomial

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 = 0$$

if and only if each $a_i = 0$, and then later says that

$$2x^2 + 5x + 3 = 0?$$

3. You always ask us to explain our thinking. I know that two fractions can be equal, but their numerators and denominators don't have to be equal. What about if $\frac{a}{b} = \frac{c}{d}$, and they are both reduced to simplest form. Does $a = c$ and $b = d$, and how should we explain this?

4. The homework assignment asked us to find the next term in the list of numbers $3, 5, 7, \ldots$? John said the answer is 9 (he was thinking of odd numbers), I said the answer is 11 (I was thinking odd prime numbers), and Mary said the answer is 3 (she was thinking of a periodic pattern). Who is right?

5. We know how to find 2^2, but how do we find $2^{2.5}$ or $2^{\sqrt{2}}$?

6. My father was helping me with my homework last night and he said the book is wrong. He said that $\sqrt{4} = 2$ and $\sqrt{4} = -2$, because $2^2 = 4$ and $(-2)^2 = 4$, but the book says that $\sqrt{4} \neq -2$. He wants to know why we are using a book that has mistakes.

7. Why should we learn the quadratic formula when our calculators can find the roots to 8 decimal places?

8. The carpenter who is remodeling our kitchen told me that geometry is important. He said he uses his tape measure and the Pythagorean theorem to tell if a corner is square. He marks off 3 inches on one edge of the corner, 4 inches on the other edge, and then connects the marks. If the line connecting them is 5 inches long, he knows by the Pythagorean theorem that the corner is square. This seems different from the way we learned the Pythagorean theorem.