Homework 3 **Analysis II** **Fall 2018**

The problems are due on Thursday, September 27.

For all students:

Problem 1 Let (A_n) be a sequence of non-empty closed bounded subsets in a complete metric space (X, d), such that $A_{n+1} \subseteq A_n$ for all $n \in \mathbb{N}$, and such that $\lim_{n\to\infty} diam\, A_n = 0$. Show that $\bigcap_{n=1}^{\infty} A_n$ consists of exactly one point.

The *diameter* of a bounded set A in a metric space is defined as follows:

$$diam\, A = \sup\{d(x, y) \mid x, y \in A\}.$$

Problem 2 Show that a metric space (X, d) is totally bounded if and only if every sequence in X contains a Cauchy subsequence.

Problem 3 Recall that a set A is called *countable* if there is a bijection $\phi : \mathbb{N} \to A$.

Show that a metric space (X, d) is compact, if and only if every countable subset $A \subseteq X$ has an accumulation point.

Problem 4

1. Let (X, d) be a compact metric space. Show that there is a countable subset $D \subseteq X$ such that $cl(D) = X$.
2. Give an example of a metric space (X, d) such that $cl(D) \neq X$ holds for every countable subset $D \subseteq X$.