Homework 6 **Analysis II** **Fall 2018**

The problems are due on Thursday, November 6.

For all students:

Problem 1 Let K be a compact subset of $\mathbb{R}$, and for all $n \in \mathbb{N}$ let $f_n : K \to \mathbb{R}$ be a continuous function. Prove or disprove: If (f_n) converges pointwise to a continuous function $f : K \to \mathbb{R}$, then (f_n) converges uniformly to f.

Problem 2 Let $f : \mathbb{R} \to \mathbb{R}$ be the limit of a uniformly converging sequence of polynomials $P_n : \mathbb{R} \to \mathbb{R}$. Show that f is a polynomial.

The last two problems investigate the validity of the commutativity law for converging real series.

Problem 3 Let (a_n) be a sequence of real numbers, such that $\sum |a_n|$ converges, and let $\rho : \mathbb{N} \to \mathbb{N}$ be any bijection. Show that

$$\sum_{n=1}^{\infty} a_{\rho(n)} = \sum_{n=1}^{\infty} a_n.$$

Problem 4 For a real number x, define $x^+ = \max\{x, 0\}$ and $x^- = \max\{-x, 0\}$.

Let (a_n) be a sequence of real numbers, such that $\sum a_n$ converges, while $\sum |a_n|$ diverges.

1. Show that the partial sums of $\sum a_n^+$ and the partial sums of $\sum a_n^-$ are unbounded increasing sequences. Note that $\lim_{n\to\infty} a_n^+ = 0$ and $\lim_{n\to\infty} a_n^- = 0$.

2. Let $L \in \mathbb{R}$. Using the result above, show that there is a bijection $\rho : \mathbb{N} \to \mathbb{N}$ such that

$$\sum_{n=1}^{\infty} a_{\rho(n)} = L.$$