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2.18

Moderator: **helmut** [Moderator Control Panel]

Author	Message

anthony.d.dennis23

offline

Math Cadet

Joined: Sat, 28 Mar 2020 15:16
Posts: 7

Post subject: 2.18 **Posted:** Tue, 31 Mar 2020 11:31

$Let(a_n)$ be an increasing bounded sequence. By the Completeness Axiom the sequence converges to some real number a, then $\forall \epsilon > 0$, $\exists N \in \mathbf{N}, s.t. for\ n \geq N,\ |a_n - a| < \epsilon$

Since $|a_n - a| < \epsilon$, it follows that the range A is bounded by $a - \epsilon < a_n < a + \epsilon$ and thus $a_n - \epsilon < a$. So $\forall \epsilon > 0\ and\ n \geq N$, a is an upper bound of A.

Since (a_n) is increasing, $a_n + \epsilon$ is always an upper bound of A, by the fact that $a - \epsilon < a_n < a + \epsilon$ it follows that $a < a_n + \epsilon$. So $\forall \epsilon > 0\ and\ n \geq N$, a is less than all other upper bounds of A.

Therefore because a is an upper bound of A and a is less than all other upper bounds of A, $a = supA$

helmut

online

Site Admin

Joined: Sat, 26 Apr 2003 15:14
Posts: 2224
Location: El Paso TX (USA)

Post subject: Re: 2.18 **Posted:** Tue, 31 Mar 2020 17:36

> **anthony.d.dennis23 wrote:**
> $Since|a_n - a| < \epsilon$, it follows that the range A is bounded by $a - \epsilon < a_n < a + \epsilon$ and thus $a_n - \epsilon < a$.

Does $|a_n - a| < \epsilon$ hold for **all** n?

The greater danger for most of us lies not in setting our aim too high and falling short; but in setting our aim too low, and achieving our mark. - Michelangelo Buonarroti

anthony.d.dennis23 **Post subject:** Re: 2.18 **Posted:** Thu, 02 Apr 2020 09:22

offline

Math Cadet

Joined: Sat, 28 Mar 2020 15:16
Posts: 7

$Let(a_n)$ be an increasing bounded sequence. By the Completeness Axiom the sequence converges to some real number a, then $\forall\epsilon > 0, \exists N \in \mathbf{N}, s.t. for\ n \geq N,\ |a_n - a| < \epsilon$

Since for n $\geq N, |a_n - a| < \epsilon$, it follows that the range A is bounded by $a - \epsilon < a_n < a + \epsilon$ and thus $a_n - \epsilon < a$. So $\forall\epsilon > 0\, and\, n \geq N$, a is an upper bound of A.

Since (a_n) is increasing, $a_n + \epsilon$ is always an upper bound of A, by the fact that $a - \epsilon < a_n < a + \epsilon$ it follows that $a < a_n + \epsilon$. So $\forall\epsilon > 0\, and\, n \geq N$, a is less than all other upper bounds of A.

Therefore because a is an upper bound of A and a is less than all other upper bounds of A, $a = supA$

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solivas11

Post subject: Re: 2.18 **Posted:** Thu, 02 Apr 2020 16:57

online

Math Cadet

Joined: Mon, 30 Mar 2020 21:12
Posts: 6

I like this. I would just add to account for $n < N$: since a_n is an increasing sequence, then $Let B = \{a_n | \forall n < N\}. Then B < A\ \forall n \in \mathbb{N}$

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helmut

Post subject: Re: 2.18 **Posted:** Sun, 05 Apr 2020 16:04

online

Site Admin

Joined: Sat, 26 Apr 2003 15:14
Posts: 2224
Location: El Paso TX (USA)

Getting there. One more improvement needed.

Easier to do contradicton for the two parts.

Suppose a is not an upper bound for A. Then there is an n such that $a < a_n$. Explain why this can't be true.

Suppose b<a. Then (why?) there is an a_n with $a_n > b$, and thus b is not an upper bound of A.

The greater danger for most of us lies not in setting our aim too high and falling short; but in setting our aim too low, and achieving our mark. - Michelangelo Buonarroti

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solivas11

Post subject: Re: 2.18 **Posted:** Mon, 06 Apr 2020 22:29

online

Math Cadet

Joined: Mon, 30 Mar 2020 21:12
Posts: 6

> **helmut wrote:**
>
> Suppose a is not an upper bound for A. Then there is an n such that $a < a_n$. Explain why this can't be true.

a_n is an increasing bounded sequence. By CA, $a_n \rightarrow\ a$

$Assume\ a \neq UB(A) \iff \neg(\forall n \in \mathbb{N}\ a \geq\ a_n) \iff \exists n \in \mathbb{N}\ a <\ a_n$

$$Let\ N_1 \in \mathbb{N}\ s.t.\ a < a_{N_1}$$

$$Let\ \varepsilon = a_{N_1} - a\ >\ 0$$

$$\exists N_2 > N_1\ s.t.\ |a_n - a| < \frac{\varepsilon}{2}\ \forall\ n \geq\ N_2$$

$$|a_{N_2} - a| < \frac{\varepsilon}{2}\ =\ \frac{a_{N_1} - a}{2}$$

$$\frac{a - a_{N_1}}{2} < a_{N_2} - a < \frac{a_{N_1} - a}{2}$$

$$\frac{a - a_{N_1}}{2} + (\frac{a + a_{N_1}}{2}) < a_{N_2} - a + (\frac{a + a_{N_1}}{2}) < \frac{a_{N_1} - a}{2} + (\frac{a + a_{N_1}}{2})$$

$$a < a_{N_2} + \frac{a_{N_1} - a}{2} < a_{N_1}$$

$$\Rightarrow a_{N_2} < a_{N_1},\ where\ N_1 < N_2$$

$$\Rightarrow \text{contradicts } a_n \text{ as increasing sequence.}$$

> **helmut wrote:**
>
> Suppose b<a. Then (why?) there is an a_n with $a_n > b$, and thus b is not an upper bound of A.

$$Let\ b < a$$

$$Let\ \varepsilon = b - a > 0$$

$$\exists N \in \mathbb{N}\ s.t.\ |a_n - a| < \frac{\varepsilon}{2}\ \forall\ n \geq\ N$$

$$\frac{-\varepsilon}{2} < a_n - a < \frac{\varepsilon}{2}$$

$$\frac{-(a-b)}{2} + (\frac{a+b}{2}) < a_n - a + (\frac{a+b}{2}) < \frac{a-b}{2} + (\frac{a+b}{2})$$

$$b < a_n - \frac{a-b}{2} < a$$

$$\Rightarrow\ b < a_n\ \forall\ n \geq\ N$$

$$\Rightarrow\ b\ \neq\ UB(A)$$

Question: Is the above in addition to Anthony's portion or is this separate?

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helmut **Post subject:** Re: 2.18 **Posted:** Tue, 07 Apr 2020 14:14

online

Site Admin

> **solivas11 wrote:**

Joined: Sat, 26 Apr 2003 15:14
Posts: 2224
Location: El Paso TX (USA)

> **helmut wrote:**
>
> Suppose a is not an upper bound for A. Then there is an n such that $a < a_n$. Explain why this can't be true.

a_n is an increasing bounded sequence. By CA, $a_n \to a$

$Assume\ a \neq UB(A) \iff \neg(\forall n \in \mathbb{N}\ a \geq a_n) \iff \exists n \in \mathbb{N}\ a < a_n$

$Let\ N_1 \in \mathbb{N}\ s.t.\ a < a_{N_1}$

$Let\ \varepsilon = a - a_{N_1} > 0$

$\exists N_2 > N_1\ s.t.\ |a_n - a| < \frac{\varepsilon}{2}\ \forall\ n \geq N_2$

$|a_{N_2} - a| < \frac{\varepsilon}{2} = \frac{a_{N_1} - a}{2}$

$\frac{a - a_{N_1}}{2} < a_{N_2} - a < \frac{a_{N_1} - a}{2}$

$\frac{a - a_{N_1}}{2} + (\frac{a + a_{N_1}}{2}) < a_{N_2} - a + (\frac{a + a_{N_1}}{2}) < \frac{a_{N_1} - a}{2} + (\frac{a + a_{N_1}}{2})$

$a < a_{N_2} + \frac{a_{N_1} - a}{2} < a_{N_1}$

$\Rightarrow a_{N_2} < a_{N_1},\ where\ N_1 < N_2$

$\Rightarrow$ contradicts a_n as increasing sequence.

> **helmut wrote:**
>
> Suppose b<a. Then (why?) there is an a_n with $a_n > b$, and thus b is not an upper bound of A.

$Let\ b < a$

$Let\ \varepsilon = b - a > 0$

$\exists N \in \mathbb{N}\ s.t.\ |a_n - a| < \frac{\varepsilon}{2}\ \forall\ n \geq N$

$\frac{-\varepsilon}{2} < a_n - a < \frac{\varepsilon}{2}$

$\frac{-(a-b)}{2} + (\frac{a+b}{2}) < a_n - a + (\frac{a+b}{2}) < \frac{a-b}{2} + (\frac{a+b}{2})$

$b < a_n - \frac{a-b}{2} < a$

$\Rightarrow\ b < a_n\ \forall\ n \geq N$

$\Rightarrow\ b \neq UB(A)$

Question: Is the above in addition to Anthony's portion or is this separate?

I made one change!
Problem is done. 1 credit each for Anthony and Sergio.

____
The greater danger for most of us lies not in setting our aim too high and falling short; but in setting our aim too low, and achieving our mark. - Michelangelo Buonarroti

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